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Steel Containment Resistance Under General Dynamic Pressures

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Prepared by L. Greimann, F. Fanous, D. Bluhm

Ames Laboratory

Prepared for
U.S. Nuclear Regulatory
Commission

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Steel Containment Resistance Under General Dynamic Pressures

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Prepared by
L. Greimann, F. Fanous, D. Bluhm

Ames Laboratory
Iowa State University
Ames, IA 50011

Prepared for
Division of Engineering
Office of Nuclear Reactor Regulation
U.S. Nuclear Regulatory Commission
Washington, D.C. 20555
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ABSTRACT

Severe accident loadings on containments can include combinations of uniform internal pressure, dynamic pressure and seismic loadings. The objective of this work is to develop a methodology for calculating the ultimate capacity of steel containments under very general global or local, uniform or spatially varying dynamic loadings. An axisymmetric geometric model is used and the circumferential variation of the loads is expressed in terms of Fourier series components. The solution, i.e., displacement and stress resultants at any point of time, are obtained and also represented by Fourier series components. Values of these components at discrete locations are evaluated using modal superposition. Failure, i.e., leakage of the containment, is defined to occur when either the membrane strain in the shell reaches twice the shell material yield strain or a bifurcation point occurs. The ASME Boiler and Pressure Vessel Codes Level C criteria can also be applied as a failure condition. A computer program called LOADS was written to evaluate the point-in-time displacements and stress resultants. This program uses the results obtained from a modified BOSOR4 modal analysis program. The results of the LOADS program are input into a modified BOSOR5 computer code to evaluate the containment resistance which considers large deformation and material nonlinear behavior. A user's manual for the BOSOR4/LOADS/BOSOR5 program set is presented. The modified BOSOR codes and the LOADS program are installed on the Brookhaven National Laboratory VAX system.

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EXECUTIVE SUMMARY

Summary

The function of the containment structure is to restrict leakage for a variety of environmental and accident loading conditions. Although very rare, loadings more severe than the design loading can be postulated. Besides uniform internal pressure, severe accident loadings on containments include dynamic and seismic loadings. The objective of this work is to develop a methodology for assessing the ultimate capacity of steel containments due to global or local, uniform or spatially varying dynamic loadings. The scope of this work is limited to unpenetrated steel containments. Failure modes of penetrations, anchorages and any other modes are not considered. Vessel resistance is predicted based upon either a nonlinear large displacement analysis or the ASME Boiler and Pressure Vessel Code Level C criteria.

The containment vessel experiences various types of loadings that vary in time and both circumferential and meridional directions. A three-dimensional finite element model could be used to analyze these variations but, in most cases, would be impractical. In this work an axisymmetric geometric model is used and the circumferential variation of the pressure is represented by a Fourier Series. The response of the containment is expressed in terms of the modal coordinates. The Wilson θ method is used to solve numerically the modal equilibrium equation. The displacements and the stress resultants at any point of time are represented by Fourier series at any point of time. Values of these components at discrete locations are evaluated using modal superposition.

Plastic collapse of the containment shell can occur in either tension or compression regions whereas bifurcation buckling is associated solely with compressive stresses, e.g., in the lower portion of the cylindrical shell during seismic loading. Consistent with the use of an axisymmetric containment model, an axisymmetric "worst meridian" prebuckled shape is used to investigate buckling. The effects of the circumferential variation of the stress resultants on the shell buckling strength are studied. All of the investigations conducted regarding this variation have concluded that the buckling stress is not sensitive to this variation if the variation is slow with respect to the buckled shape. The "worst meridian" approach is found to yield to a conservative value of the shell buckling strength. Shell vibration and buckling analyses are performed using modified BOSOR4 and BOSOR5 finite difference codes, respectively. These programs were chosen over the other programs since they are available to the NRC staff on the Brookhaven National Laboratory computer facilities. BOSOR5 results are calibrated with experimental results. Geometric imperfections are modeled in the meridional direction by a cosine deviation from the perfect shape. No imperfections in the circumferential direction were included. Other nonaxisymmetric factors such as penetrations cannot be modeled in the BOSOR5 axisymmetric model. However, several experimental investigations are cited to show that penetrations with diameters less than $2\sqrt{rt}$ and reinforced in accordance with the ASME replacement rules do not affect the shell buckling load. A quasi-static solution summing the applied dynamic and inertial pressures is used. These pressures' input into the BOSOR5 model are equivalent pressures that produce the correct stress resultants along the "worst" meridian under consideration. Some changes are introduced into the current BOSOR5 version to accommodate point wise variation of these pressures. The new versions of BOSOR4 and BOSOR5 programs are referred to as B4AMES and B5AMES, respectively.

Failure of the containment (leakage) is defined to occur when the maximum membrane strain in the shell reaches twice the yield strain. This strain will be reached by loading along a fundamental loading path unless compressive stresses introduce a bifurcation point. It is assumed that, if a bifurcation point is reached, buckling will occur along a secondary path and the strain will exceed twice the yield valve along this path. The ASME Boiler and Pressure Vessel Codes Level C criteria is also applied as a failure condition. The ASME membrane stress intensity is calculated at each mesh point for specified meridians at each time. The pressure peak which produces the ASME Level C allowable stress intensity is determined.

The analysis methodology was implemented into a computer code called LOADS. This program evaluates the equivalent axisymmetric point-in-time pressures and the point-in-time stress intensities at specific meridians using B4AMES modal analysis output and the nonaxisymmetric applied dynamic pressure. The equivalent pressures are used as an input to the B5AMES analysis. Several verification problems were analyzed to check the results of this methodology with those from other analyses. The number of the meridional and circumferential modes of vibration in the analysis has a significant effect on the accuracy of the results. A simplified containment vessel was also analyzed to demonstrate the application of the outlined methodology.

A User's Manual for the LOADS program and the modified BOSOR programs is presented. Samples of input and output as well as the files that will be created during the execution of the B4AMES, LOADS and B5AMES codes are discussed. The attached sample input data are typical of those required to analyze a containment structure.

Conclusions

The approach outlined in this work can be used to assess the ultimate capacity of steel containments under global or local dynamic loads. The methodology could assist the Nuclear Regulatory Commission in their regulatory and licensing activities regarding severe accident loadings. The computer program written to apply this approach to a containment vessel is easy to follow. The user can evaluate containment vessel resistance based on either plastic collapse bifurcation buckling or Level C Code criteria.

Recommendations

Perhaps the biggest shortcoming of the analysis procedure is that the user must prepare two separate input files for the B4AMES and B5AMES programs. Future work should be devoted to writing one preprocessor that can be used for both programs. Also, nonlinear dynamic behavior should be incorporated. This requires significant effort but some analysis techniques are available. Additionally, of course, there is always the problem of the buckling of thin shells that could use more study. The containment vessel strength under combined seismic and pressure loadings should also be examined. Further studies should investigate the relationship of buckling to leakage.

1. INTRODUCTION

1.1 Background

Many organizations and investigators have become involved in assessing the ultimate strength of steel containments under internal uniform static pressure [1.1]. Most of this work has been the result of continued interest by the Nuclear Regulatory Commission (NRC) in the response of nuclear power plant systems to severe accidents. Besides uniform internal pressure, severe accident loadings on containments also include dynamic pressures (time and spatially varying) or seismic forces. A unified analytical treatment of these loadings, individually and combined, would assist the NRC in their regulatory and licensing activity regarding severe accidents.

1.2 Objective and Scope

The objective of this work is to develop a single methodology for assessing the ultimate capacity of steel containments due to rather general pressure loadings - static and dynamic, global and local, uniform and spatially varying.

The scope of this project is limited to steel containments and:

- pressure loads only (no seismic forces);
- shell failure modes only (no penetration or anchorage modes).

Additionally, for reasons explained in appropriate places of the text, a linear dynamic analysis is performed to predict inertial forces (Section 2) and nonlinear shell buckling predictions are based upon an axisymmetric prebuckled shape (Section 3). Ultimate capacity will be defined by either strain and buckling criteria or by ASME Boiler and Pressure Vessel Code Level C stress criteria (Section 4). The methodology will be implemented into computer programs and validated by comparison with experimental data (Section 5).

1.3 Selection of Analysis Method

The assessment of the ultimate capacity of a containment implies a structural analysis which includes nonlinear behavior. A range of methods are available to perform this analysis from a pr/t, single degree of freedom type approximation to a complete three dimensional finite element shell model with material and geometric nonlinearities. An argument can be made for each model within this range, depending upon the accuracy required and the time and funds available. Considering the importance of the current problem, it seems reasonable to use an analysis method which is as sophisticated as possible. This would tend toward the three-dimensional shell model. However, funds and time were not available within this project to permit this type of analysis. Additionally, this approach does have some technical shortcomings--it has not been thoroughly correlated with experiment and certain model parameters, e.g., geometric imperfections are not readily available. For these reasons and others mentioned in Section 3.1, an intermediate level analysis will be used herein. This model will consist, basically, of an axisymmetric approximation to the containment shell. Hence, containment geometry, imperfections and material properties must be assumed to remain constant around the circumference.

The axisymmetric containment model forces certain restrictions in the containment analysis:

- Nonsymmetric loadings can only be analyzed if the response is linear. The Fourier decomposition of the applied loading and the subsequent superposition of results is valid only if the response is linear. Section 2 will summarize the linear analysis of the containment under nonsymmetric dynamic loads. The extrapolation of the inertial forces from this analysis into the inelastic regime are discussed in Section 3.8.

- Shell bifurcation buckling will be investigated only from an axisymmetric prebuckled shape. Hence, the nonsymmetric stress resultants obtained from the dynamic analysis must be assumed to be axisymmetric for the buckling analysis. Section 3.3 discusses the validity of this "worst" meridian approach.

- Nonsymmetric features such as openings and imperfection must be handled approximately. Sections 3.3.3 and 3.4 discuss this approximation.

Each of these simplifications will be made with an eye toward obtaining a lower bound selection of the structural resistance.

Section 5 summarizes the analysis method and its implementation into computer programs including:

- B4AMES - A modified version of the BOSOR4 program to compute the mode shapes and natural frequencies of the axisymmetric shell model.

- LOADS - A program which integrates the modal equations of motion and computes the equivalent axisymmetric dynamic pressures (a linear dynamic analysis).

- B5AMES A modified version of the BOSOR5 program to check the inelastic buckling resistance of the axisymmetric model using the "worst" meridian approximation.

A demonstration example and verification problems follow.

1.4 References

1.1 Greimann, L., Fanous, F. and Bluhm, D., "Containment Analysis Techniques, A State-of-the-Art Summary," Report to Sandia National Laboratory, October 1983.

2. NONSYMMETRIC DYNAMIC ANALYSIS

The containment vessel experiences various types of loadings. Many of these loads are time dependent, such as those that result from loss of coolant accident (LOCA) or safety relief valve blowdown. Additionally, these pressures are not uniformly distributed around the shell circumference, θ, and along the meridian, s (see Fig. 2.1). In these cases, a dynamic analysis should be used to evaluate the containment strength. Such an analysis will be accomplished using a dynamic finite difference analysis.

2.1 Modal Analysis

As discussed in Sec. 1.3 of this work, the containment will be analyzed as an axisymmetric shell with nonsymmetric dynamic loads. For such a case, it is common to represent the circumferential variation of the input and response by a Fourier series in the circumferential coordinate θ [2.1, 2.1, 2.3], i.e,

$$\begin{aligned} u(s,\theta,t) &= \sum \bar{u}_{an} \cos n\theta + \sum \bar{u}_{bn} \sin n\theta \\ v(s,\theta,t) &= -\sum \bar{v}_{an} \sin n\theta + \sum \bar{v}_{bn} \cos n\theta \\ w(s,\theta,t) &= \sum \bar{w}_{an} \cos n\theta + \sum \bar{w}_{bn} \sin n\theta \end{aligned} \tag{2.1}$$

in which u, v, and w are the shell meridional, circumferential and normal displacements, respectively. The Fourier coefficients, $\bar{u}_{an}$, $\bar{u}_{bn}$, etc., are functions of time, t, and the meridional coordinate, s. Discretization in the meridional direction is by usual finite element (or difference) methods, i.e., polynominal interpolation functions. The resulting dynamic equilibrium equations are

$$[M]\{\ddot{u}(t)\} + [C]\{\dot{u}(t)\} + [K]\{u(t)\} = \{F(t)\} \tag{2.2}$$

in which $\{u(t)\}$ are discrete nodal values of the Fourier coefficients. The mass, damping and stiffness matrices are [M], [C], and [K], respectively. The nodal forces $\{F(t)\}$ are work equivalent to the applied pressure $p(s,\theta,t)$

$$p(s,\theta,t) = \sum \bar{p}_{an} \cos n\theta + \sum \bar{p}_{bn} \sin n\theta \tag{2.3}$$

Because of the orthogonality of the Fourier series components and the linearity of the problem, Eq. 2.2 uncouples for the individual Fourier components, e.g., only $\bar{u}_{an}$, $\bar{v}_{an}$, $\bar{w}_{an}$ and $\bar{p}_{an}$ for a single value of n will be involved in a particular set of equations.

There are at least two different approaches to the solution of Eq. 2.2. The first procedure involves step-by step time integration. The second alternative is to use the modal superposition method [2.3, 2.4, 2.5, 2.6]. One of the principle advantages of this approach is that the response of the system is expressed by the first m modes of vibration. On the other hand, this technique requires several matrix operations which are major computational efforts not required in the direct integration procedure, e.g., eigenvalve extraction. This approach is adopted in this work.

The first m mode shapes and natural frequencies of the system are obtained by solving for the eigenvalues, ω_i, and the eigenvectors $\{\phi_i\}$ from the undamped homogeneous equation

$$\omega_i^2 \, [M]\{\phi_i\} + [K]\{\phi_i\} = \{0\}, \; i = 1, \ldots\ldots m \qquad (2.4)$$

The response of the system at time, t, can be expressed in terms of the modal coordinates, $\{\lambda(t)\}$, as

$$\{u(t)\} = [\Phi]\{\lambda(t)\} \qquad (2.5)$$

where $[\Phi]$ is a rectangular (n by m) matrix which contains the m mode shapes as columns. Substituting Eq. 2.5 into Eq. 2.2 and introducing the modal damping in the usual manner [2.4] yields an uncoupled set of differential equations for $\{\lambda\}$ as

$$\{\ddot{\lambda}(t)\} + [2\xi\omega]\{\dot{\lambda}(t)\} + [\omega^2]\{\lambda(t)\} = \{F_\Phi(t)\} \qquad (2.6)$$

where

$$[2\omega\xi] = \begin{bmatrix} 2\omega_1\xi_1 & & & \\ & 2\omega_2\xi_2 & & \\ & & \ddots & \\ & & & 2\omega_m\xi_m \end{bmatrix} \qquad (2.7)$$

$$[\omega^2] = \begin{bmatrix} \omega_1^2 & & & \\ & \omega_2^2 & & \\ & & \ddots & \\ & & & \omega_m^2 \end{bmatrix} \qquad (2.8)$$

$$\{F_\Phi(t)\} = [M_\Phi]^{-1} \, [\Phi]^T \, \{F(t)\} \qquad (2.9)$$

$$[M_\phi] = [\phi]^T [M] [\phi] \qquad (2.10)$$

in which ξ_i is the damping ratio of mode i. Notice that each uncoupled equation for λ_i in Eq. 2.6 represents a single degree of freedom system characterized by the undamped frequency, ω_i, and a damping ratio, ξ_i.

2.2 Time Integration

Several numerical integration techniques are available to solve the uncoupled differential equations given in Eq. 2.6. The solution of Eq. 2.6 will be accomplished here using the Wilson - θ method [2.6]. The basic idea of the Wilson - θ method is to assume a linear variation of the acceleration, $\ddot{\lambda}$, from time t to $t + \theta\Delta t$, where $\theta > 1.0$. This assumption leads to a parabolic and cubic variation of the velocity $\dot{\lambda}$ and the displacement λ, respectively.

For unconditional stability, a value of $\theta > 1.37$ should be employed. In this work, the value of 1.4 recommended in Ref. 2.6 is used. Additionally, to predict the dynamic response of the structure accurately in the highest mode, the time step Δt should be selected corresponding to the smallest natural period of interest, T_s, in the system. A time step size equal to $T_s/10$ will be employed to perform the numerical integration.

2.3 Stress Resultants

As with the displacements and applied pressures, the resulting stress resultants are represented by Fourier series [2.1, 2.2] which, for example, for the meridional or circumferential stress resultants, are

$$N(s,\theta,t) = \sum \overline{N}_{an} \cos n\theta + \sum \overline{N}_{bn} \sin n\theta \qquad (2.11)$$

in which $\overline{N}_{an}$ and $\overline{N}_{bn}$ are the Fourier coefficients and are functions of t and s. Values of these components at discrete nodal locations are

$$\{N(t)\} = [N_\phi] \{\lambda(t)\} \qquad (2.12)$$

where $[N_\phi]$ are nodal values of the Fourier coefficients for the modal stress resultants.

2.4 References

2.1 Gallagher, R.H., Finite Element Analysis Fundamentals, Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1975.

2.2 Bushnell, D., "Plastic Buckling of Various Shells," Joint ASME/ASCE Mechanics Conference, Boulder, CO, June 22-29, 1981.

2.3 Boyce, W.E. and R.C. Diprima, Elementary Differential Equations and Boundary Value Problems, New York, John Wiley and Sons, 1977.

2.4 Clough, R.W. and J. Penzien, Dynamics of Structures, New York, McGraw-Hill, 1975.

2.5 Briggs, J.M., Introduction to Structural Dynamics, New York, McGraw-Hill, 1964.

2.6 Bathe, K.J. and E. L. Wilson, Numerical Methods in Finite Element Analysis, Englewood Cliffs, New Jersey, Prentice-Hall, Inc., 1976.

3. NONLINEAR QUASI-STATIC ANALYSIS

Plastic collapse and bifurcation buckling are potential failure modes of the containment shell. Consistent with the use of an axisymmetric containment model, an axisymmetric prebuckled shape will be used to investigate buckling (Sec. 1.3). The stress resultants at several meridians are obtained by the dynamic analysis in the previous section. A buckling analysis is performed for each meridian by assuming that the stress resultants do not vary circumferentially. The "worst" meridian is the one which gives the lowest predicted buckling load. The remainder of this section is devoted to an explanation of this approach.

3.1 The Shell Buckling Problem

Shell buckling is not a well understood phenomenon. The analysis of the elastic (and inelastic) buckling strength of thin shells has long frustrated structural engineers. Two classic problems--that of an axially compressed thin circular cylinder and that of a sphere under external pressure--which appear on the surface to be simple, have proven to be extremely complex. For these cases, the buckling load predicted by classic, small displacement elastic theory is well above observed experimental results (by 3 to 5 times). It is beyond the scope of the current work to provide a complete review of the state-of-the-art in this area, but the reader is referred to [3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7] for good summaries of the topic.

Many explanations and theories have been advanced to reconcile the difference between prediction and reality. There is now general agreement that the discrepancy is due to "imperfections" in the shell, i.e., deviations of the shell from the ideal. These imperfections include geometric imperfections (deviation from a perfect shape), material imperfections (residual stress), and boundary condition imperfections (geometric constraint conditions, applied load eccentricities). Undoubtedly these parameters are of random nature and are not well defined at the time of design. Remedies for the discrepancy between theory and experiment involve varied approaches ranging from theoretical models that include the imperfection (as-built shapes and materials, weld and fabrication residual stresses), to empirical "knockdown factors" or safety factors. Typically the former are too expensive and the latter are too crude. However, a theoretical model enables the analyst to include the imperfection parameters (if known). Most current approaches represent a combination of these two extremes.

Before proceeding to consideration of various shell analysis techniques, a conceptual description of the instability behavior of shells is appropriate. (This follows [3.1 and 3.2]). Figure 3.1 schematically illustrates the possible load-displacement behavior of a shell, e.g., a cylindrical shell under axial load. Loading may progress along a fundamental path OBA. The deformed shape remains essentially the same along this path with only the magnitude of the displacements increasing, e.g., axisymmetric shape for axial tension or compression, first Fourier component for pure bending. Eventually, an instability occurs along this path, point A, at a point of maximum load. This point is variously referred to as the plastic collapse load, limit point buckling or "snap-through". For this behavior, the "failed" deflected shape differs from the initial deflected shape only in magnitude.

For many shell configurations and loadings, e.g., axial compression of a cylindrical shell, an alternative equilibrium path is available. That is, a bifurcation point B exists at which two equilibrium shapes are possible. At this point, the shell will follow the path of least energy, path BC. This behavior is termed bifurcation buckling. The post-buckling path is called a secondary path. Deformations along this path differ completely from the pre-bifurcation path, e.g., asymmetric buckling of an axially compressed cylinder in which wrinkles or lobes form around the circumference. Elastic buckling occurs if the shell is elastic at the bifurcation point, i.e., point B is below point Y. If B is above Y, inelastic (or plastic) buckling occurs.

The purpose of shell analysis is to predict the load displacement behavior, i.e., Path OBA or OBC in Fig. 3.1. No post-buckling analyses will be performed in the current work. For almost all shell problems, the post-buckling path is unstable (as shown) and the structure has reached its maximum loads. For certain structures, e.g., flat plates with in-plane compression and ellipsoidal heads with internal pressure, the post-buckling path is stable. It will here be assumed that the post-buckling path is unstable and carries the shell well into the inelastic range, i.e., to strains several times the yield value. This is a conservative assumption. For example, for the case of elastic buckling, strains may remain below yield and the structure could return to its undeformed state upon removal of the load.

Current shell buckling analysis techniques can be (loosely) subdivided into two types:

- Design type equations in an analytical form with appropriate corrections for imperfections [3.5 (Papers No. 10, 16, 17, 18), 3.8]. Typically these equations are for special, but common, cases, e.g., stiffened and unstiffened cylinders, cones, spheres.

- Numerical (finite element type) approaches which can handle more general shell shapes. Unfortunately, these solutions must also be empirically corrected for imperfections and are very expensive [3.3, 3.4, 3.5 (Papers No. 5, 19), 3.6, 3.9].

Because of its generality, the finite element type approach will be presented here.

Within the finite element solutions, two alternatives are again available [3.1, 3.3, 3.6, 3.9, 3.10]:

- A complete three-dimensional model of the shell using two-dimensional shell elements. This model can be used to model both meridionally and circumferentially varying imperfections and loads. Other nonaxisymmetric features such as penetrations and circumferentially varying nonlinearities can also be included in the model.

- An axisymmetric model of the shell with one-dimensional axisymmetric shell elements. Only axisymmetric imperfections and loads may be modeled.

From an analytical point of view, the first choice is preferable. This method has, to date, been used primarily in a research mode for analyzing experimental models with carefully measured (known) imperfections; for localized analyses, e.g., openings and penetrations; or for numerical experimentation with random imperfections [3.9, 3.10]. Results have been interesting and useful, but not necessarily more accurate than those for the simpler axisymmetric model. From a practical point of view, the first approach is quite expensive, and the authors agree with [3.10] that the second is, at this time, a better choice.

In the remainder of this section guidelines for the buckling analysis of nominally axisymmetric shells with an axisymmetric model will be presented. Approximate incorporation of nonaxisymmetric features will be emphasized. The effect of this approximation on the plastic collapse strength and the elastic buckling strength will be discussed. The BOSOR5 [3.10] computer code will be used for the axisymmetric analysis.

3.2 BOSOR5 Program

The BOSOR5 [3.11] program is a finite difference code for the analysis of complex branched shells of revolution. This program was developed from the BOSOR4 [3.12] computer code and is designed for axisymmetric and nonaxisymmetric buckling analyses resulting from axisymmetric loading only. The program has the capability to handle nonlinear material behavior which was not considered in BOSOR4.

BOSOR5 has been used worldwide and is considered error free due to its extensive use. The program can handle segmented and branched axisymmetric shells with discrete ring stiffeners, meridian discontinuities, and multimaterial wall construction. The shell wall can be made up of as many as six layers, each made of a different nonlinear material. In the prebuckling analysis, moderately large deflection, axisymmetrically deformed prebuckled states are calculated. Realistic engineering details such as interior supports may be modeled. In addition to plastic material behavior, creep may be accounted for. The program also has restart and graphics capability.

The BOSOR5 program is chosen over other programs for several reasons. First of all it is available to the NRC staff on the Brookhaven National Laboratory computer facilities. Also, it is not general purpose, but rather is designed only for shells of revolution. Therefore, the input parameters are relatively few. Secondly, the extensive use of the program improves its reliability. The BOSOR5 program is perhaps the most widely recognized buckling program for axisymmetric shells. Also, rings and stiffeners are relatively easy to model. In addition, an interactive version of the program is available on the Digital Corporation VAX mini-computer.

3.3 Nonaxisymmetric Loads

Many loadings for shells of revolution are not axisymmetric, i.e., the shell stress resultants vary around the shell circumference as in the case of bending. In this case, the meridional, circumferential, and normal displacements, respectively, can be represented by summations of terms of the form (see Eq. 2.1):

$$\begin{aligned} u_n &= \overline{u}_n \sin n\theta \\ v_n &= \overline{v}_n \cos n\theta \\ w_n &= \overline{w}_n \sin n\theta \end{aligned} \tag{3.1}$$

in which n corresponds to the circumferential wave number in the prebuckled shape. When BOSOR5 is being used for the buckling analysis, the circumferential variation of the stress resultants is not permitted. Buckling of nonsymmetrically loaded cylindrical shells can be handled as suggested in Ref. [3.4, 3.13]. The prebuckled stress distribution is first calculated as the superposition of several Fourier circumferential harmonics as in Sec. 2. This is the prestress solution for the nonsymmetric load at all values of the longitudinal and circumferential coordinates. The buckling load is approximated by assuming that the membrane stress distribution along the "worst" meridian controls. In this approach the longitudinal and circumferential stress resultants which are calculated on the "worst" meridian are taken to be uniformly distributed around the entire shell circumference, i.e., axisymmetric. Several meridians may have to be examined to locate the "worst" case, i.e., the case with the lowest buckling load. Several authors agree that, if the predominate stress resultants vary slowly in the circumferential direction with respect to the buckling wave, the buckling stress is not sensitive to the nonsymmetry of the loading [3.3, 3.4, 3.5, 3.6, 3.9, 3.10, 3.14, 3.15]. They also agree that, if it does vary rapidly the "worst" meridian is, at least, conservative. From an intuitive view, it appears reasonable that uniform compression around the entire circumference is a "worse case" than a circumferentially varying compression with the same maximum value. The following summarizes some previous studies that considered the stability of axisymmetric shells under nonaxisymmetric stress states, as related to the "worst" meridian approach.

3.3.1 Elastic Buckling of Nonaxisymmetrically Loaded Perfect Shells

3.3.1.1 Longitudinal Stress Resultants

The elastic buckling of a long cylindrical shell with an axial compressive stress resultant over a small circumferential arc length (see Fig. 3.2.(a)) is reported in Ref. [3.15]. The results are summarized in Fig. 3.2.(b). The figure illustrates the ratio of critical stress of the nonaxisymmetric case, N_{cr}, to the critical stress for uniform axial compression, $\overline{N}_{cr}$,

$$\frac{N_{cr}}{\overline{N}_{cr}} = \frac{\text{critical stress, nonaxisymmetric case}}{\text{critical stress, axisymmetric case}} \tag{3.2}$$

plotted versus a nondimensional arc length. The critical stress, N_{cr}, is always equal to or greater than the critical stress of the same shell under axisymmetric axial compression. The increase in N_{cr} is noticeable only if the loaded arc length is small.

The elastic stability of circular cylindrical shells under bending ($n = 1$ for prebuckled shape) has been investigated by Seide and Weingarten [3.16] as a bifurcation buckling problem. They concluded that the critical buckling stress for bending of a shell with an r/t ratio of 100 is about 1.5 percent higher than the critical buckling stress under uniform axial load. For thinner shells this difference is even smaller. This result is well above the critical stress found by Brazier [3.17] who included the effect of the shell cross-section ovalization on the stability of long cylinders under pure bending. If a long cylinder is subjected to bending, its cross-section flattens and, consequently, the bending stiffness deteriorates with increasing moment. However, it appears from experiments on the bending of thin shells that the shells fail through buckling in a wavy pattern, similar to shells under axial compression, rather than through collapse by flattening of the cross-section. The restraint at the ends of the tested shells is a contributing factor. Hence, a finite length shell will buckle at load levels higher than these predicted by Brazier.

BOSOR4 has been used to study the variation of the elastic buckling stress of stiffened shells with circumferentially varying longitudinal stress resultants [3.9]. To study a nonaxisymmetric effect with an axisymmetric model, the authors of [3.9] modeled a cylinder of infinite length as a portion of a torus with a large radius R_t (see Fig. 3.3). As R_t approaches infinity, the torus approaches a straight cylinder [3.9, 3.14]. The cross-section need not be circular nor the thickness constant. Loading can vary around the circumference. Since the torus is a shell of revolution, the BOSOR programs can be used without any special alterations. The loading considered in [3.9] is similar to that in Fig. 3.2(a), i.e., a uniform longitudinal stress resultant is applied over the arc length, b. The results are summarized in Fig. 3.4 where the ratio $N_{cr}/\overline{N}_{cr}$ is plotted against the ratio of the loaded arc length b to the arc length of the buckled wave for the uniform compression case, $\overline{b}$ (see Fig. 3.5'. These results are shown for the case of a ring and stringer stiffened shell (smeared stiffeners, general buckling), a shell with stringers only (discrete stringers, general buckling), and a stringer stiffened shell with a restricted axial buckled wavelength (discrete stringer and rings, local buckling between rings).

These results illustrate that the elastic buckling stress of a shell subjected to circumferentially varying longitudinal stress does not differ significantly from that of uniformly loaded shell if the variation is not rapid with respect to the buckled shape. Hence, if the longitudinal stress resultant is approximately uniform over one-half a buckled wavelength, the circumferential variation of stress can be ignored. In this regard, a "worst" meridian axisymmetric analysis will produce acceptable results. If the circumferential variation is rapid, the results will, at least, be conservative.

3.3.1.2 Circumferential Stress Resultants

Circumferential variations may also have an effect for situations in which the circumferential stress dominates. The effect of a circumferentially varying radial pressure [Fig. 3.6(a)] on the critical elastic buckling stress is shown in Fig. 3.6(b) [3.18]. By comparing the dashed curves (uniform pressure case) to the solid (cosine variation of pressure) one can see that the uniform case always gives a slightly lower critical stress.

Several cylindrical and conical shells heated along an axial strip of varying width have been analyzed [3.13]. It was found that a cylinder or a cone heated along a narrow strip will buckle at approximately the same stress as a uniformly heated shell. This result applies even if the half-wavelength of the buckled shape is approximately the same as the circumferential extent of the heated region. Although this project is not concerned with temperature loading, this example helps to demonstrate the validity of the "worst" meridian approach.

The conclusion, then, for nonaxisymmetric circumferential stress resultants is similar to that obtained for shells with nonaxisymmetric longitudinal stress resultants, i.e., the circumferential variation of stress resultants does not significantly affect the elastic buckling stress if the variation is slow with respect to the buckled shape. In general, the elastic buckling stress for a nonaxisymmetrically loaded shell is conservatively approximated by an axisymmetric model. This is the basis of the "worst" meridian approach.

3.3.1.3 Shear Stress Resultants

BOSOR5, as currently configured, does not include shear stress resultants even though such cases may be axisymmetric, e.g., torsion. Hence, the program cannot be used to detect the elastic buckling strength for cases with shear stress resultants or for cases with combined normal and shear stress resultants. This does not appear to be a severe limitation because, in many cases, the shear stress is not significant, e.g., planes of symmetry. The user is warned, however, that, in the most general case, shear stress, in combination with other stress resultants, reduces the buckling capacity [3.19].

3.3.2 Plastic Collapse of Perfect Shells

As discussed in Sec. 3.1 and illustrated in Fig. 3.1, a shell may reach its maximum load without changing its basic deformation pattern, i.e., plastic collapse. Again it can be reasoned that the "worst" meridian approach is conservative. The case of a cylinder subjected to pure bending is instructive. If no elastic buckling occurs (elastic buckling was discussed in the previous section), the "worst" meridian approach predicts that the maximum stress resultant for bending is equal to that for pure axial load, i.e., N_x of $F_y t$ or a maximum moment of $\pi r^2 t F_y$. If a complete plastic hinge is permitted to form, the maximum moment would be $4/\pi$ times this. If strains of $2\varepsilon_y$ are permitted at the extreme fiber, the moment is 3.636 $r^2 t F_y$ or 1.16 times the "worst" meridian value.

For other nonsymmetric loading cases, such analytical justification of the conservativeness of the "worst" meridian was not readily available. However, it seems clear that, in the general case, the stress resultants for plastic collapse in a nonsymmetric mode will be larger than for a symmetric mode. In a nonsymmetric collapse mode, the (material and/or geometric) instabilities are limited to less than a full circumference. The remainder of the circumference is stable and provides support to, restricts the instability of, and allows a redistribution around the unstable portion. From only an intuitive point of view, the stress resultants required to produce this instability will be greater than for the case in which the instability engulfs the entire circumference.

Alternatively, the above arguments also demonstrate that, for a given set of stress resultants, the shell strains will be greater if the stress resultants are axisymmetric than if they have only a local maximum. Again, as an example, consider a cylinder with a stress resultant equal to $F_y t$ and an elastic-perfectly plastic material. If this stress resultant results from an axial load, the strains are very large. If the stress resultant is due to an applied moment, the maximum strain is equal to the yield value and occurs only at two extreme fibers.

As mentioned in Sec. 3.3.1.3, BOSOR5 does not incorporate membrane shear stress. In the case of plastic collapse, this means the effect of shear stress on the von Mises yield surface is not included. Thus, as for elastic buckling, BOSOR5 can only be used for those cases in which shear stresses are not significant, e.g., planes of symmetry.

In summary, the above discussion indicates that the "worst" meridian approach can also be used for plastic collapse. It will be conservative if the shear stress is not a significant portion of the von Mises effective stress, i.e., less than 20 percent of the normal stress.

3.3.3 Imperfect Shells

As previously mentioned, differences between the first-order theoretical and the measured buckling strengths are usually ascribed to imperfections. In the case of real structures which contain unavoidable imperfections, there is no such thing as the true bifurcation buckling illustrated in Fig. 3.1. Instead, the structure will follow a path below OBC with a maximum load below Point B. How much below B depends on the imperfection sensitivity of the shell [3.1, 3.2]. For the sphere under external pressure and a cylinder under axial compression, the reduction is large. For the imperfect shell case, the distinction between plastic collapse and bifurcation buckling is less clear. The deformed shape at maximum load can be similar to the prebuckled shape as the perfect bifurcation buckled shape or the dominate imperfection mode or a combination of these.

If it were possible to predict all of the imperfection patterns resulting from the fabrication process, one could conceivably evaluate the critical load of such a structure with much greater certainty. A three-dimensional finite element model would be appropriate [3.9] in which the nonsymmetric imperfections can be explicitly included. An alternative is to utilize an axisymmetric model with an approximate axisymmetric imperfection, as would be appropriate for a BOSOR5 analysis [3.1, 3.3].

A nominally axisymmetric shell has circumferential and longitudinal imperfections. Imperfections sympathetic to the various buckling modes are considered to be the most important [3.1, 3.4, 3.5 (Paper No. 19), 3.6]. Thus, longitudinal imperfections tend to have the most effect on longitudinal buckling strength and circumferential imperfections on circumferential strength [3.3, 3.5 (Paper No. 7, 9, 15)]. As suggested by several researchers [3.1, 3.3, 3.4, 3.6 (Paper No. 5, 6, 19)], the longitudinal imperfection can be modeled as a cosine wave in the meridional direction, s, i.e.,

$$w_{imp} = w_o\left(1 - \cos \frac{2\pi s}{L_o}\right) \tag{3.3}$$

where w_o is the radial imperfection amplitude (one-half the peak-to-trough amplitude of the imperfection). In the design case when w_o is not known, it can be estimated in terms of the usual fabrication tolerances [3.20] as with

$$w_o = 0.0025\ \gamma\ D_{nom} \tag{3.4}$$

with

$$\gamma = (D_{max} - D_{min})/0.01\ D_{nom} \tag{3.5}$$

in which D_{max} and D_{min} are the maximum and minimum diameters of the imperfect shape, and D_{nom} is the average diameter. For axial compression of unstiffened shells which buckle nonsymmetrically, Koiter [3.1, 3.7] has suggested that the imperfection wavelength L_0 be taken as equal to the wavelength for axisymmetric buckling of a perfect cylinder, which is [see Fig. 3.7(a)]

$$L_K = 3.5\sqrt{rt} \tag{3.6}$$

Gellin [3.21] gave a similar expression to evaluate the critical wavelength for cylindrical shells stressed beyond the proportional limit. For stiffened shells, Ref. [3.1] recommends that the wavelength of the axisymmetric imperfection be determined as follows: the maximum inward imperfections are taken to coincide with the rings if the rings are spaced at a distance less than 1.5 times the length L_K [see Fig. 3.7(b)]. However, for rings spaced at greater distances, the axial wavelength is taken to be half the ring spacing [see Fig. 3.7(c)]. In order to phase the imperfection with the ring spacing, especially for variable spacings, slight modifications will be required in BOSOR5.

Circumferential imperfections cannot be included directly in the axisymmetric model. Ref. [3.2] suggests that cylinders subject to radial pressure are imperfection insensitive. However, cylinders with hydrostatic pressure are imperfection sensitive because the pressure on the ends produces axial loads to which cylinders are notoriously imperfection sensitive. If this is true, circumferentially varying imperfections need not be included in the model since they are not generally sympathetic to the axial compression buckling mode. Bushnell [3.1] suggests that one can account for the effect of nonaxisymmetric imperfections on longitudinal buckling by dividing the critical load, based upon an axisymmetric imperfection pattern, by the ratio of the local radius of curvature at a "flat spot" to the nominal radius. He observes, however, that the standard deviation of the ratio of theory to test is not noticeably reduced when this is done.

3.3.4 Equivalent Axisymmetric Loads

The approach described above requires that stress resultants on the "worst" meridian be assumed axisymmetric and examined for buckling and collapse. BOSOR5 can only accept axisymmetric pressures and line loads, not stress resultants. Hence, the stress resultants must be converted to equivalent axisymmetric pressures. For an axisymmetric shell, two of the governing differential equations for nonsymmetric analysis are [3.7]

$$\begin{aligned}
&\frac{\partial(rN_1)}{r\partial s} + \frac{\partial N_{12}}{r\partial\theta} - \frac{N_2\cos\phi}{r} + \frac{1}{rr_1}\left[-\frac{\partial(rM_1)}{\partial s} + \frac{\partial M_{12}}{\partial\theta} + M_2\cos\phi\right] = m\ddot{u}-p_u \\
&-\frac{\partial(\partial(rM_1))}{r\,\partial s^2} - \frac{\partial^2 M_2}{r^2\,\partial\theta^2} + \frac{\partial(M_2\cos\phi)}{r\,\partial s} + \frac{2\partial^2(rM_{12})}{r^2\,\partial\theta\partial s} - \frac{N_1}{r_1} - \frac{N_2}{r\sin\theta} = m\ddot{w}-p_w
\end{aligned} \tag{3.7}$$

in which N_1, N_2 and N_{12} are the meridional, circumferential and shear stress resultants, respectively and M_1, M_2 and M_{12} are the corresponding moment resultants. The displacements u and w are meridional and normal, respectively, m is the mass per unit area and other quantities are defined in Fig. 3.8. The acceleration terms on the right represent inertial pressures or, by D'alemberts' principle, the negative of equivalent static pressures. The applied pressures are p_u and p . For axisymmetric response, N_{12} and M_{12} and derivatives with respect to θ are zero. An axisymmetric analysis (BOSOR5) will reproduce the meridional and circumferential stress resultants on the "worst" meridian if the following pressures are used for the nth harmonic (Eq. 3.1).

$$p^e_{u_n} = \left(\bar{p}_{u_n} - m\ddot{\bar{u}}_n - \frac{n\bar{N}_{12n}}{r} - \frac{n\bar{M}_{12n}}{r_1 r}\right) \sin n\theta$$

$$p^e_{w_n} = \left(\bar{p}_{w_n} - m\ddot{\bar{w}}_n + \frac{n^2 \bar{M}_{2n}}{r^2} - \frac{2n}{r^2}\frac{\partial (r\bar{M}_{12n})}{\partial s}\right) \sin n\theta \qquad (3.8)$$

where the super bar refers to the amplitude of the Fourier harmonic as in Eq. 3.1. The "worst" meridian is located at the angle θ. If a modal analysis is being used, the displacements and stresses are decomposed into their modal coordinates (Sec. 2). For mode j, the equivalent modal pressures are:

$$p^e_{u_j} = \left(\omega_j^2\, m\, \bar{\Phi}_{u_j} - \frac{n\bar{N}_{12_j}}{r} - \frac{n\bar{M}_{12_j}}{r_1 r}\right) \sin n\,\theta$$

$$p^e_{w_j} = \left(\omega_j^2\, m\, \bar{\Phi}_{w_j} + \frac{n^2\bar{M}_{2_j}}{r^2} - \frac{2n\,\partial (r\bar{M}_{12_j})}{r^2\,\partial s}\right) \sin n\,\theta \qquad (3.9)$$

in which ω_j is the modal frequency, $\bar{\Phi}_u$ and $\bar{\Phi}_w$ are the meridional and normal modal displacements, respectively, and $\bar{N}_{12_j}$, $\bar{M}_{12_j}$ and $\bar{M}_{2_j}$ are the modal stress resultants (Eq. 2.12). The applied and inertial pressures combine to give the first term on the right if the ratio p/m is expressed in the modal coordinates. The total equivalent pressures are obtained by superposition of the modal pressures:

$$p^e_u = \sum_j p^e_{u_j}\,\lambda_j$$

$$p^e_w = \sum_j p^e_{w_j}\,\lambda_j \qquad (3.10)$$

where λ_j are the modal coordinate (Sec. 2). These equations, as well as Eq. 3.1, show only the nonsymmetric terms of the Fourier series in θ. A complete series also involves the symmetric terms (Eq. 2.1).

3.4 Openings in Shells

A cylinder with a penetration cannot be analyzed with a BOSOR5 axisymmetric model. It is known that if the penetrations are reinforced by the ASME area replacement rule [3.22], the strength of a penetrated shell under uniform internal pressure will exceed that of an unpenetrated shell. In general, the ASME rules require that the area removed for the opening should be replaced by an equal area of reinforcement around the opening circumference. In this situation yielding is the usual failure mode and, as the stresses are redistributed, the reinforcement participates fully in the plastic behavior. This argument can also be applied to the general case of plastic collapse.

Numerous investigations have been conducted to determine the effects of a penetration and its reinforcement on the elastic and inelastic buckling resistance of the shell [3.23, 3.24, 3.25]. In all of these studies, experimental cylindrical models with various ratios of reinforcement area to cutout area were tested under axial compression. Figure 3.9, which summarizes the results, illustrates that the area replacement rule does not necessarily restore the full unpenetrated buckling strength of the shell. In Ref. [3.3] it is concluded, however, that small holes (diameter less than $2\sqrt{rt}$) have less effect than other imperfections and that, for this case, the area replacement reinforcement adequately restores the strength.

An analytical study has been conducted by Meller and Bushnell [3.26] to calculate the strength of the models tested in [3.25]. This was accomplished with three-dimensional finite element models which were analyzed using the STAGSC [3.27] computer code. The unpenetrated specimen was analyzed with and without including the measured initial imperfections. The analytical collapse load for the perfect shell was nine percent greater than the analytical collapse load for the imperfect shell. The average experimental collapse load for two specimens was one percent higher than the calculated load for the imperfect cylinder. No measured imperfections were reported in [3.25] for the shells with openings; hence, perfect analytical models were used in the analysis of the penetrated shells. Figure 3.10 is a plot of the ratio of the analytical calculated buckling load of the perfect penetrated shell to that of the unpenetrated perfect shell as a function of the opening radius, a, divided by $\sqrt{rt}$.

The discrepancy in the results reported in [3.23] and [3.24] makes it difficult to establish specific conclusions concerning the buckling resistance of penetrated shells in relation to unpenetrated shells.

The results given in [3.25] show that openings in stiffened shells will reduce the buckling load, even though the penetrations are reinforced in accordance with the ASME rules. However, the buckling strength of an axisymmetric shell is not affected by reinforced openings with diameters less than $2\sqrt{rt}$. Larger penetrations, such as equipment hatch openings, could reduce the shell buckling resistance, even if they are reinforced as required by ASME rules. However, a good first approximation of the effect of the reinforced opening on the buckling strength can be obtained by an axisymmetric analysis. The penetration and its reinforcement would be assumed to be axisymmetric about the axis of the penetrating cylinder and the containment shell would be approximated by a segment of an equivalent spherical shell. The relative effect of the reinforced opening can be studied by comparing the buckling strength of this approximation to that of the unpenetrated equivalent spherical shell. (Note, this approximation should not be used to determine absolute buckling strength but only to determine relative buckling strength of the penetrated vs. unpenetrated shells.) In many cases, a three-dimensional finite element analysis is recommended to evaluate the shell critical load, similar to that in [3.26]. Each large penetration must be handled on an individual basis.

3.5 Quasi-static Approximation

Shell structures often experience loads that are functions of time. In this case, the problem should be analyzed as a dynamic buckling problem in which the inertial forces from the dynamic loading interact with the inertial forces from the buckling motions. A usual procedure for dynamic loading is to perform a dynamic analysis and compare the dynamic stresses at each time ("point-in-time or freezing-in-time stresses") to the static buckling stresses [3.6], i.e., to neglect buckling inertial forces. Several authors have recognized that this is a conservative approach [3.4, 3.6, 3.10]. Research is currently under way [3.19] to study the effect of this assumption on cases with seismic loading.

With the "freezing-in-time" approach, the dynamic problem can be analyzed as the quasi-static problem suggested in Sec. 3.3.4, i.e.,

$$p_s = p_d - m\ddot{u} \qquad (3.11)$$

in which the quasi-static pressure, p_s, is the algebraic sum of the applied dynamic pressure, p_d, and the inertial pressure, $- m\ddot{u}$. (Note that Eq. 3.8 includes additional terms which convert the pressure to an equivalent axisymmetric value.)

If the inertial pressures are correct, i.e., obtained from an appropriate dynamic analysis, the quasi-static solution indicated in Eq. 3.11 should involve no approximation. Thus for example, it would precisely follow the time behavior and reproduce the correct cyclic (hysteretic) response history.

BOSOR5 nicely handles the solution of Eq. 3.11 for axisymmetric shells. The pressures p_s can be input into BOSOR5 as a function of the time parameter to follow the quasi-static pressure history produced by Eq. 3.10 and the time integration in Sec. 2. (A slight modification to BOSOR5 was necessary to permit the flexibility necessary for this input--see Sec. 5.3).

3.6 Other BOSOR Model Guidelines

3.6.1 Mesh Guidelines

The BOSOR programs are based on the finite difference energy method to analyze smooth or stiffened shell structures. The shell and the ring stiffeners can be modeled for the finite difference analysis using shell segments. A discrete ring option is available in the BOSOR programs for the idealization of the circumferential stiffeners. A maximum of 25 shell segments can be used to model a structure. Each segment is divided into a number of finite difference elements using at least five mesh (nodal) points with a maximum of 95 points. In calculating buckling loads, the BOSOR manual recommends using more than four mesh points per half wavelength of the buckling mode. The number of these mesh points in the segment is one of the most important variables in the analysis. This governs, to a large extent, the accuracy of the solution. It is recommended that the distance between the mesh points be restricted to $\sqrt{rt}/2$. This also satisfies the recommended mesh point spacing for the buckling analysis.

3.6.2 Plasticity Model

BOSOR5 [3.11] includes options for both the deformation and flow theories of plasticity. Even though the flow theory is analytically more attractive, buckling test results tend to agree better with deformation theory analysis [3.3, 3.4, 3.11 (Vol. 3, p. 50), 3.28]. Deformation theory will be used here.

3.6.3 Convergence Criteria

To accomplish the nonlinear (material and geometry) solution, an iterative approach is used with the BOSOR programs. The procedure is to increase the applied load by small increments and allow the program to iterate until it converges to a final solution. In BOSOR4 and 5, the geometric nonlinearity solution, is said to be converged if the ratio of the change in the displacement vector to the current total displacement vector is less than a specified value of 0.001 [3.11]. The nonlinear material problem is handled in BOSOR5 using a similar technique. In this case, the solution is said to be converged when the plastic strain increment becomes less than 0.1 percent [3.11].

3.7 BOSOR5 Calibration

One advantage of the BOSOR5 program is its widespread use. Hence, results have been compared to experimental results in many cases [3.28, 3.29, 3.30, 3.31]. Much of the applicable work has been summarized in Table 3.1 and can be used in calibrating BOSOR5 results. In Table 3.1, the shell and loading type for the experimental work are identified. Deformation plasticity theory was used for all analyses. The specimens were aluminum and steel. Imperfections were not included in the torispherical and elipsoidal head analysis since these heads were not considered sensitive to imperfections [3.29, 3.30]. The stiffened cylinders with external pressure were machined and had small imperfections, which were not included in the BOSOR5 model [3.31]. Axisymmetric imperfections were included in the analysis in [3.1, 3.3] and [3.5, No. 5] as discussed in Sec. 3.3.2. The other cylinders from [3.11] were relatively thick ($r/t < 100$) and imperfection insensitive. The mean and standard deviations of the ratio of experimental result to BOSOR5 result are listed. As one would expect, the mean and standard deviation of this ratio vary noticeably with experimentation. The results shown in Table 3.1 yield a mean and coefficient of variation of the ratio of the experimental to BOSOR5 results of one and 0.135, respectively.

3.8 Nonlinear Extrapolation

As mentioned in Sec. 1.3 and detailed in Sec. 4, nonlinear behavior of the containment is permitted before failure. However, because of the restrictions imposed by the axisymmetric shell model (Sec. 1.3), a nonlinear, nonsymmetric analysis will not be attempted herein. In lieu of such an analysis, it is appropriate to extrapolate the results of a linear analysis into the nonlinear region. This, of course, is not generally permitted [3.32]. Typically, this type of extrapolation is acceptable only if the nonlinear deformed shape is "regular" and does not have any localized instabilities such as local strain concentrations or strength discontinuities. In the following, we propose the extrapolation of linear pressures. That is, we propose to perform a linear dynamic analysis and obtain the quasi-static pressures in Eq. 3.11. These "linear" pressures will be statically applied in a nonlinear static analysis.

It is instructive to compare the results of such an approximate analysis to the exact results for an elastic-plastic single degree of freedom system. For such a system, the governing differential equation for the displacement u_p is:

$$m \ddot{u}_p + R(u_p) = F(t) \qquad (3.12)$$

in which the restoring force is

$$R(u_p) = \begin{cases} k\, u_p & u_p < u_y \\ R_m & u_p > u_y \end{cases} \qquad (3.13)$$

and u_y is the displacement at yield. This system will reach a finite maximum displacement, $u_{p\ max}$, if the long term final value of $F(t)$ is less than R_m, e.g., if $F(t)$ eventually goes to zero.

The differential equation for an elastic system with displacement u_e is:

$$m \ddot{u}_e + k u_e = F(t) \tag{3.14}$$

Extrapolating the "linear" forces into the nonlinear region, as suggested above, produces the quasi-static problem

$$R(u'_p) = F(t) - m \ddot{u}_e = k u_e \tag{3.15}$$

in which u'_p is an approximation to the nonlinear solution. For the case when the system goes beyond yield

$$k u_{e_{max}} > R_m \quad \text{if} \quad u_{e_{max}} > u_y \tag{3.16}$$

in which $u_{e_{max}}$ is the maximum displacement predicted by the elastic analysis. The quasi-static solution will, thus, produce the result that $u'_{p_{max}}$ goes to infinity. Thus, for an elastic-perfectly plastic system

$$u_{p_{max}} < u'_{p_{max}} \tag{3.17}$$

that is, the approximate solution bounds the actual response.

Another interesting case is a general nonlinear single degree of freedom system (Fig. 3.11) subjected to an impulsive load, i.e., an nonzero initial velocity, v_o. By energy considerations, the maximum displacement of this system may be found from

$$\int_o^{u_{p_{max}}} R(u)\, du = 1/2\, m\, v_o^2 \tag{3.18}$$

Note that, if the system is strain softening, i.e.,

$$\frac{d^2 R(v)}{d\, u^2} < o \tag{3.19}$$

we can write (see Fig. 3.11)

$$\frac{u_{p_{max}}\, R(u_{p_{max}})}{2} < \int_o^{u_{p_{max}}} R(u)\, du < 1/2\, k\, u_{pmax}^2 \tag{3.20}$$

and, since R(u) is less then ku,

$$\frac{R^2(u_{p_{max}})}{2\,k} < \int_0^{u_{p_{max}}} R(u)du < 1/2\;k\;u^2_{pmax} \qquad (3.21)$$

For the corresponding elastic system, energy balance requires

$$1/2\;k\;u^2_{e_{max}} = 1/2\;m\;v_0^2 \qquad (3.22)$$

Eq. 3.18 and Eq. 3.21 combine with Eq. 3.22 to show

$$\frac{R(u_{p_{max}})}{k} < u_{e_{max}} < u_{p_{max}} \qquad (3.23)$$

The approximate nonlinear solution u_p' from the extrapolated linear forces will be (Eq. 3.15):

$$R(u'_{p_{max}}) = k\;u_{e_{max}} \qquad (3.24)$$

which, from Eq. 3.23, gives

$$R(u_{p_{max}}) < R(u'_{p_{max}}) \qquad (3.25)$$

Since R is montonically increasing until the maximum

$$u_{p_{max}} < u'_{p_{max}} \qquad (3.26)$$

which is like Eq. 3.17, i.e., the approximate solution is an upper bound to the exact.

For the general loading case, no proof exists that the approximate solution is an upper bound. In fact, cases can be found for which this is not the case - depending upon the slope of the resistance function and the time variation of the force. For multi-degree-of-freedom systems, no bounds exist unless the system can be approximated by a single degree of freedom system, i.e., by a Rayleigh Ritz procedure. This is reasonably possible only if the dynamic shape of the system does not change significantly with time, i.e., if the nonlinear behavior is "uniformly" distributed thoughout the structure or if any local concentrations of strain and displacement do not significantly affect the overall shape. In a building, such would be the case, for example, if only one column at a given floor level has an excursion into the nonlinear range during seismic response. In this case, the deformations of the column would be controlled by the surrounding elastic columns and the overall dynamic shape would remain essentially linear. If, on the other hand, all the columns at one level become nonlinear, the deformed shape will be drastically different from the linear shape and the extrapolation of linear forces is meaningless.

In summary, an approximate nonlinear solution will be obtained in this work by extrapolating the linear pressures. There is no positive assurance that this approach is conservative. This approach will still be followed, however, on the basis that: (1) the extrapolation is not particularly extreme, i.e., to a ductility, at most, of two; (2) the nonlinear behavior is, most likely, localized and controlled by the surrounding linear behavior; and, (3) any unconservatism is probably offset by other conservatisms, e.g., the "worst" meridian approach. This approach is not inconsistent with the ASME Code which allows an elastic analysis for stresses beyond the yield strength for certain cases with limited plasticity [3.22, Paragraph NE 3227.6].

3.9 Summary

Many shells of revolution structures experience loadings (static and/or dynamic) that produce nonaxisymmetric stress resultants which may cause instability. Guidelines for using BOSOR5 to calculate the buckling stress resultants for nonaxisymmetrically loaded axisymmetric shells are presented.

A circumferential variation in the stress resultants is not permitted in the BOSOR5 axisymmetric model. Several investigations have concluded that the buckling stress is not sensitive to variations which are small with respect to the buckling wave. Therefore, the buckling strength can be conservatively evaluated assuming that the membrane stresses along the "worst" meridian are axisymmetric, if shear stresses are not significant.

Geometric imperfections have a significant effect on the shell buckling load. In the BOSOR5 axisymmetric model, an effective imperfection can be modeled as a cosine wave in the meridional direction.

Other nonaxisymmetric factors such as penetrations cannot be included in the BOSOR5 axisymmetric model. Experimental investigations have concluded that penetrations with diameters less then $2\sqrt{rt}$ and reinforced in accordance with the ASME area replacement rules do not affect the shell buckling load.

For shells with dynamic loads, a quasi-static solution summing applied and inertial pressures is appropriate. Extrapolation of these quasi-static pressures into the nonlinear regime is not necessarily conservative but is justified on the basis that only mild nonlinearities are permitted in the failure criteria.

3.10 References

3.1 Bushnell, D., "Elastic-Plastic Buckling of Axially Compressed Ring Stiffened Cylinders--Test Versus Theory," Welding Research Council, Bulletin 282, November 1982.

3.2 Chajes, A., "Post Buckling Behavior," Journal of the Structural Division, 109 (10), ASCE, pp. 2450-2462, Oct. 1983.

3.3 Bushnell, D., "Plastic Buckling of Various Shells," Joint ASME/ASCE Mechanics Conference, June 21-29, 1981, Boulder, CO.

3.4 Bushnell, D., "Buckling of Shells--Pitfall for Designers," AIAA Journal, 19 (9), pp. 1183-1226, Sept. 1981.

3.5 Buckling of Shells in Offshore Structures, Ed. by Harding, J.E., Dowling, P.J. and Agelidis, N., London: Granda Publishing, 1982, selected papers.

3.6 Seide, P., Weingarten, V. and Masri, S., "Buckling Criteria and Application of Criteria to Design of Steel Containment Shells," Report to NRC, NUREG/CR-0793, May 1979.

3.7 Brush, D.O. and Almroth, B.O., Buckling of Bars, Plates, and Shells, Chapter 7, New York: McGraw Hill, 1975.

3.8 "Shells (Buckling)," Chapter 8, Engineering Journal, American Institute of Steel Construction, Second Quarter, pp. 102-111, 1982.

3.9 Almroth, B., Rankin, N. and Bushnell, D., "Buckling of Steel Containment Shells, Task 3: Parameter Studies," Report to the U.S. NRC, NUREG/CR-2836, Vol. 3, December 1982.

3.10 "Chicago Bridge and Iron's comments on NUREG CR-7093, 'Buckling Criteria and Application of Criteria to Design of Steel Containment Shells', May 1979," available from NRC, Washington, November 1979.

3.11 Bushnell, D., "BOSOR5 Program for Buckling of Elastic-Plastic Complex Shells of Revolution Including Large Deflections and Creep," Structural Mechanics Laboratory, Lockheed Missiles and Space Co., Inc., Palo Alto, CA, 1974.

3.12 Bushnell, D., "BOSOR4 Program for Stress, Buckling and Vibration of Complex Shells of Revolution," Structural Mechanics Laboratory, Lockheed Missiles and Space Co., Inc., Palo Alto, CA, 1975.

3.13 Bushnell, D. and Smith, S., "Stress and Buckling of Nonuniformly Heated Cylindrical and Conical Shells," AIAA Journal, 9 (12), pp. 2314-2321, Dec. 1971.

3.14 Bushnell, D., "Stress Buckling and Vibration of Prismatic Shells," AIAA Journal, 9 (10), pp. 2004-2013, Oct. 1971.

3.15 Hoffs, N.J., Chao, C. and Madsen, W.A., "Buckling of a Thin-Walled Circular Cylindrical Shell Heated Along an Axial Strip," Journal of Applied Mechanics, pp. 253-258, June 1964.

3.16 Seide, P. and Weingarten, V.I., "On the Buckling of Circular Cylindrical Shells Under Pure Bending," Journal of Applied Mechanics, Vol. 28, pp. 112-116, 1961.

3.17 Brazier, L.G., "On the Flexure of Thin Cylindrical Shells and Other Thin Sections," Proc. Roy. Soc., Series A., Vol. 116, pp. 106-115, 1926.

3.18 Almroth, B.O., "Buckling of a Cylindrical Shell Subjected to Non-uniform External Pressure," Journal of Applied Mechanics, 29, pp. 675-682, Dec. 1962.

3.19 Baker, W.E. and Bennet, J.G., "Experimental Investigation of the Buckling of Nuclear Containment-like Cylindrical Geometries under Combined Shear and Bending," distributed at Structural Mechanics in Reactor Technology, Chicago, August, 1983, and submitted to Nuclear Engineering and Design Journal.

3.20 ASME Boiler and Pressure Vessel Code, Sec. III, Class MC Components, Subsection NE, Paragraphs 4221.1 and 4221.2, 1980.

3.21 Gellin, S., "Effect of an Axisymmetric Imperfection on the Plastic Buckling of Axially Compressed Cylindrical Shell," Journal of Applied Mechanics, 46, pp. 125-131, March 1979.

3.22 ASME Boiler and Pressure Vessel Code, Sec. III, Class MC Components, Subsection NE, Paragraphs 3331-3335, 1980.

3.23 Miller, C.D., "Experimental Study of Buckling of Cylindrical Shells with Reinforced Openings," ASME/ANS Joint Conference, Portland, July 26-28, 1982.

3.24 Bennett, J.G., Dove, R.C. and Butler, T.A., "An Investigation of Buckling of Steel Cylinders with Circular Cutout Reinforced in Accordance with ASME Rules," Report to the U.S. NRC, NUREG/CR-2165, June 1982.

3.25 Bennett, J.G. and Baker, W., "Buckling Investigation of Ring-Stiffened Cylindrical Shell with Reinforced Openings Under Unsymmetrical Axial Loads," Report to the U.S. NRC, NUREG/CR-2966, Oct. 1982.

3.26 Meller, E. and Bushnell, D., "Buckling of Steel Containment Shells, Task 2: Elastic-Plastic Collapse of Nonuniformly Axially Compressed Ring-Stiffened Cylindrical Shell with Reinforced Openings," Report to the U.S. NRC, NUREG/CR-2836, 2, December 1982.

3.27 Almroth, O.B. and Brogan, F.A., "The STAGS Computer Code," NASA CR-2950, Feb. 1978.

3.28 Bushnell, D., "Elastic-Plastic Buckling of Internally Pressurized Ellipsoidal Pressure Vessel Heads," Welding Research Council, Bulletin 267, May 1981.

3.29 Lagae, G. and Bushnell, D., "Elastic-Plastic Buckling of Internally Pressurized Torispherical Vessel Heads," Nuclear Engineering and Design, 48, pp. 405-414, 1978.

3.30 Bushnell, D. and Gallety, G.D., "Stress and Buckling of Internally Pressurized, Elastic-Plastic Torispherical Vessel Heads - Comparison of Test and Theory," Journal of Pressurized Vessel Techniques, 99, pp. 39-53, Feb. 1977.

3.31 Bushnell, D., "Buckling of Elastic-Plastic Shells of Revolution with Discrete Elastic-Plastic Rings," International Journal of Solids and Structures, 12, pp. 51-66, 1976.

3.32 Clough, R.W. and Penzien, J., Dynamics of Structures, McGraw Hill, 1975.

4. FAILURE CRITERIA

4.1 Membrane Strain Criteria

The purpose of the containment vessel is to prevent the release of radioactivity in the event of an accident within it. To perform this function, the containment must provide a leaktight barrier. Failure of containment is considered to occur when radioactive leakage of the containment occurs which exceeds the boundary dose limits. Leakage will occur when a defect or crack passes through the entire plate thickness of the pressure retaining boundary -- the containment shell or piping or the equipment attached to the shell. Crack propagation through the containment shell can be predicted, to a limited degree, by fracture mechanics principles [4.1]. However, leakage of all of the other associated equipment, penetrations and piping due to containment deformation is much more difficult, if not impossible, to predict. The approach adopted herein and in previous work [4.2, 4.3] is to assume that failure (leakage) will occur in the shell or the attachments when deformations (displacements and/or strains) become "large". Or, in a different way, leakage will not occur if deformations are "small". The definition of "large" is obviously subjective.

The question of whether to use displacement or strain as a basis for defining failure is certainly debatable [4.4]. The ASME Code has provisions that allow the use of either or both [4.5, 4.6, 4.7, etc.]. On the one hand, highly localized bending strains may not be indicative of structural failure. These strains are often self-limiting, and, for usually ductile materials, have little effect on the collapse strength of the vessel. On the other hand, strains certainly are more indicative of material distress than displacement. Hence, strains (or stress) are what cause material to separate and allow leakage, whereas displacements represent material failure only insofar as they represent the integrated effect of strains. In many but not all cases, e.g., static, elastic-perfectly plastic behavior, the distinction between strain and displacements as a measure of failure is insignificant because displacements and strains become "large" at the same load. In this work, strain will be selected as a basis for the failure criteria. More particularly, the maximum membrane strain in the axisymmetric containment vessel

$$\varepsilon_m = \text{maximum membrane strain} \qquad (4.1)$$

will be used as the basis for defining failure. This follows a recommendation by Gerdeen [4.4, p. 44] that membrane strain is a better measure of distress than highly localized bending (surface) strains.

Ductility is defined as the ratio of strain capacity of the material at leakage, ε_c, to the material yield strain ε_y, that is,

$$\lambda_c = \frac{\varepsilon_c}{\varepsilon_y} \quad . \qquad (4.2)$$

As a lower limit, the ductility capacity of a vessel with a uniform static pressure is probably two.

$$\lambda_c \text{ (lower bound)} = 2 \qquad (4.3)$$

Previous (1974) ASME Codes [4.9] used this value with a factor of safety to define the limiting case for uniform static internal pressure. The current ASME Code [4.5] recommends the half linear slope method which defines the limiting static pressure, $p_{2\phi}$, as the pressure at which strains are twice the elastic strain at the same pressure. For the typically elastic-almost perfectly plastic case, deformations (strain and displacement) increase very rapidly in the vicinity of static collapse and there is no significant difference between the plastic pressure defined by Eq. 4.3 and the half linear slope method. The lower limit of two for λ_c has been suggested as a design basis by others, for example [4.4, pp. 44,51] and [4.1, p. 529]. At this small strain, the probability of failure by fracture is quite remote [4.2, p. 25; 4.4, Ch. 14].

An upper limit to the ductility capacity is, reasonably,

$$\lambda_c \text{ (upper bound)} = 10 \qquad (4.4)$$

The current ASME Code allows a maximum inelastic membrane strain, as predicted by an inelastic analysis, of one percent for elevated temperature applications (λ_c about 10) [4.7]. Gerdeen [4.4, pg. 70] recommends strain limits in the range of one to two percent if yielding rather than bursting is the criterion of the design. Other suggestions are as high as 0.5 $\varepsilon_u/\varepsilon_y$ for bursting [4.9, p. 312, 4.4, p. 68] where ε_u is the ultimate strain from a simple tension test.

The selection of a reasonable ductility capacity between the two limits of Eqs. 4.3 and 4.4 is, within the current state of knowledge, a matter of judgment. Fracture mechanics approaches lend insight into what factors affect the ductility capacity: material fracture toughness (K_{Ic}, critical stress intensity factor, or K_{Id}, dynamic critical stress intensity factor); size of defects in the material; temperature (nil-ductility transition and operating temperature); loading rate (static or dynamic); strain (or stress) gradient (uniform or highly localized peak strain). It is beyond the scope of this work to explore all these avenues. The study of a failure criteria to define leakage is a major study in itself. Fracture mechanics state-of-the art has not yet advanced to the level required to solve the problem, especially in the case of the ductile behavior of thin plates at normal temperatures. For example, the parameters discussed above are not well defined for the current problem.

The problem, therefore, remains and a failure criteria must (arbitrarily) be selected. For a smooth, machined, "perfectly fabricated" axisymmetric test specimen, a ductility near the upper bound, or even higher, could be rationalized. For a field welded in-place vessel fabricated from rolled steel plate, a lower value would be selected [4.1, p. 529].

Penetrations also present a problem. The above strain criteria is based on the maximum membrane strain from an axisymmetric analysis. Localized strains near penetrations and other strain concentrations could easily be three or four times this value. Hence, a ductility demand of ten on the axisymmetric strain could imply a local ductility requirement of thirty. (The authors admit that the correct theoretical, not practical, approach here would be to analyze each strain concentration area individually with locally defined ductility demands and capacities.) A similar nonsymmetric consideration is that even an initially "perfect" axisymmetric shell will fail by locally "bulging" along a relatively small circumferential arc length. The strains in this local bulge will be significantly larger than the average circumferential strain.

Global displacements are also a factor. Large displacements will grossly distort the shell and could, thereby, introduce failure into other components. For example, pipes and attachments at points removed from the containment may leak. Gaskets around hatches and other seals could leak due to distortion. Bellows and expansion devices have limited travel. An indirect way of limiting these displacements (particularly the radial displacement) is by limiting circumferential strain.

Considering all the effects mentioned above, for the present, it is appropriate to obtain a conservative estimate of the containment strength. The ductility capacity will be selected as two.

$$\lambda_c = 2 \tag{4.5}$$

Ductility capacity for dynamic loading can be significantly different from the static case since fracture toughness decreases with increased loading rate. Other material properties such as yield strength and modulus increase with loading rate. Larger ductility capacities are typically associated with loads of short duration, i.e., one time loads applied for a very short time. For example, allowable ductilities for impact loading [4.9, p. 312] and seismic [4.10, p. 511] are significantly higher than those for continuous, sustained loading. However, to retain the conservative character of this work, the ductility capacity values given in Eq. 4.5 will be used.

In many load cases, compressive stresses will be induced which may cause instability of the shell--geometric instability (elastic buckling) or geometric and material instability (inelastic buckling). For containments, such cases occur for torispherical and ellipsoidal heads under internal pressure and for shells under lateral loads such as seismic forces. A general discussion of plastic collapse and buckling (the fundamental and secondary loading paths) is given in Sec. 3.1 and the associated Fig. 3.1. No post-buckling analysis will be performed in the current work. It will be assumed that the post-buckling behavior is inelastic and follows a secondary path similar to BC in Fig. 3.1 [4.3]. That is, it will be assumed that, at the bifurcation point B, a buckling mechanism is formed and the membrane strains go beyond yield and reach a ductility of two in the post buckling region. This is a conservative assumption. For example, in the case of elastic buckling, strains may remain below yield in the post-buckling region.

In summary, failure is defined to occur when the maximum membrane strain reaches twice the yield strain along the fundamental load path (plastic collapse) or when a bifurcation point is reached (buckling). In the second case, the membrane strains are assumed to go beyond twice yield during post-buckling. The implementation of this criteria is discussed in Sec. 5.

4.2 ASME Level C

The ASME Boiler and Pressure Vessel Code, Section III, Division 1, [4.11] NE 3220, has established allowable stresses intensities for various load combinations. For loadings which have a very low probability of occurrence such as the dynamic overpressure conditions studied in this work, Level C stress intensity limits are frequently considered to be applicable. A summary of these requirements is given here.

Definitions

P - stress intensity (Difference between the algebraically largest principal stress and the algebraically smallest principal stress, twice the maximum shear stress.)

P_M - general primary membrane stress intensity (Average stress across an entire section of a vessel, not self-limiting. Gross deformation occurs if this stress exceeds yield. An example is general membrane stress in a cylinder or sphere with internal pressure.)

P_M' - membrane state of stress associated with the buckling analysis, S_{cr}. (Typically, a general membrane state very similar to P_m but dependent upon the type of buckling analysis.)

P_L - local primary membrane stress intensity. (Typically, self limiting like a secondary stress but redistribution takes place only after large deformations. An example is the local membrane stress near a gross structural discontinuity such as shell intersections at the springline or at a penetration.

P_B - primary bending stress intensity (Same as P_M except bending stress. An example is at the center of a flat plate with lateral pressures.)

S_y - yield stress, ASME Table I-2.0

S_{mc} - allowable stress intensity, ASME Table I-10.0

S_{cr} - Critical buckling stress (Obtained from either (1) rigorous analysis including imperfections, nonlinearities, large deformations and inertial forces: (2) classical linear analysis reduced by margins which reflect difference between theoretical and actual buckling loads ("knockdown factors"); or (3) physical tests.

Other definitions, such as secondary and peak stresses, are not necessary here since Level C does not have a requirement for these stresses.

For Level C conditions, the allowable stress intensities limit the applied stress intensities by:

Paragraph NE-3221.1

$$P_M < S_c = \text{Maximum } (1.2\ S_{mc},\ S_y) \qquad (4.6)$$

Paragraph NE-3221.2

$$P_L < 1.5\ S_c \qquad (4.7)$$

Paragraph NE-3221.3

$$P_L + P_B < \text{Minimum } (1.5,\ 2.5\text{-}1.5 \frac{P_L}{S_y})\ S_c \qquad (4.8)$$

Paragraph NE-3222.2

$$P_M' < 1.2\ S_{cr}/3 \qquad (4.9)$$

if the structure is integral and continuous and has a solid rectangular cross section. No Level C criteria are prescribed for secondary and peak stresses, e.g., local stresses near penetrations. Paragraph NE-3227.6 indicates an elastic analysis shall be performed, even though some of the allowables are beyond the linear range. The stability criteria may not be required by NRC rulemaking [4.12].

Some interpretation of the intent of Paragraphs NE-3221.2 and NE-3221.3 are necessary. Apparently the factors in Paragraph NE-3221.3(b) (1) allow for plastic redistribution in bending as a rectangular cross section goes from initial yielding to fully plastic (Paragraph NE-3221.3(d) and [4.13]). Hence, the factor 1.5 arises because the fully plastic bending capacity for a rectangular section is 1.5 times the moment at first yield and the factor 2.5-1.5 P_L/S_y accounts for the reduction in the fully plastic moment capacity as the membrane stress increases beyond 2/3 S_y. The result of a literal application of these equations is that the local primary membrane stress intensity, P_L, is limited to S_y when P_B is very small. On the other hand, when P_B is zero, Paragraph NE-3221.2 allows P_L to reach 1.5 S_y. The apparent intent of these two sections is that, if bending is required to maintain equilibrium then the reduction in plastic moment capacity caused by membrane stress must be included. If any bending which exists is not required to maintain equilibrium, a redistribution of membrane forces is permitted, i.e., P_L is permitted to go up to 1.5 S_y.

According to Table NE-3217.1, typical steel containments have only one area of primary bending stress - at the crown. Bending near the springline, basemat, shell segment intersections, penetrations and ring stiffeners is classed as secondary bending. At the crown, the primary membrane stress is classed as general. The primary membrane stress at gross discontinuities such as the springline, base mat, shell segment intersections, penetrations and ring stiffness may be classed as local if it does not exceed 1.1 P_M beyond a distance $\sqrt{rt}$ from a single discontinuity. Two such discontinuities may not exist within 2.5 $\sqrt{rt}$. This seems to imply, for example, that if rings are farther apart than 2.5 $\sqrt{rt}$, the stress between them should be classed as general. This is true for almost all containments.

Since for containment steels S_y is larger than 1.2 S_{mc}, the three stress requirements are satisfied if

$$P_M < S_y \tag{4.10}$$

$$P_L < 1.5\ S_y \quad \text{if} \quad P_L > 1.1\ S_y \text{ within } \sqrt{rt} \tag{4.11}$$

$$P_L + P_B < \text{Minimum}\left(1.5,\ 2.5 - 1.5\ \frac{P_L}{S_y}\right) S_y \tag{4.12}$$

According to the definition

$$P_M \text{ or } P_L = \text{Maximum}\left(|\sigma_1|,\ |\sigma_2|,\ |\sigma_1 - \sigma_2|\right) \tag{4.13}$$

in which σ_1 and σ_2 are the principle membrane stresses, i.e.,

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_\theta}{2} \pm \left(\left(\frac{\sigma_x - \sigma_\theta}{2}\right)^2 + \tau_{x\theta}^2\right)^{1/2} \tag{4.14}$$

in which σ_x, σ_θ and $\tau_{x\theta}$ are the meridional, circumferential, and shear membrane stresses for the axisymmetric containment.

The buckling requirement of ASME can also be variously interpreted [4.14]. Consistent with the definition, S_{cr} is the expected critical buckling stress under actual conditions. Hence, if Method (1) or Method (2) is used, material and geometric nonlinearities must be included and S_{cr} reduced by a factor of 2.4 for Level C (Eq. 4.9).

Alternatively, ASME Code Case N-284 [4.15] states that the Level C factor of safety is 1.67 for a Method (2) evaluation. The more conservative factor of 2.4 will be used here. However, the reader should note this is extremely conservative, especially for thick shells. These shells buckle inelastically at stresses near the yield stress. In this case, the Level C allowable compressive stress would be about $S_y/2.4$ which is well below the intended value of S_y.

4.3 References

4.1 Rolfe, S.T. and Barson, J.M., Fracture and Fatigue Control in Structures, Applications of Fracture Mechanics, New Jersey: Prentice Hall, 1977.

4.2 Greimann, L., et al., "Reliability Analysis of Containment Strength," Report to U.S. NRC, FIN No. A4131, NUREG/CR-1891, IS-4753, July 1982.

4.3 Greimann, L., et al., "Reliability Analysis of Steel Containment Strength," NUREG/CR-2442, June 1982.

4.4 Gerdeen, J.C., "A Critical Evaluation of Plastic Behavior Data and A Unified Definition of Plastic Loads for Pressure Components," Draft of final report submitted to Pressure Vessel and Research Committee, Welding Research Coundil, Aptil 9, 1979.

4.5 ASME Boiler and Pressure Vessel Code, Section III, Division 1, Par. II-1430, 1980.

4.6 ASME Boiler and Pressure Vessel Code, Section III, Division 1, Appendix F, 1980.

4.7 ASME Boiler and Pressure Vessel Code, Code Cases N-47-11 (1592-11), Appendix T, 1980, p. 164.

4.8 ASME Boiler and Pressure Vessel Code, Section III, Division 1, Par. ii-1430, 1974.

4.9 Structural Analysis and Design of Nuclear Plant Facilities, American Society of Civil Engineers, Manual No. 58, 1980.

4.10 Newmark, N.M. and Rosenblueth, E., Fundamentals of Earthquake Engineering, Prentice Hall, 1971.

4.11 ASME Boiler and Pressure Vessel Code, Section III, Division I, Subsection NE, 1980.

4.12 Gov, P.F. and Love, J.E., "Pressure Carrying Capacity of the Containment Structural System of the MARK III Standard Plant," in Proceedings of the Workshop on Containment Integrity, NUREG/CP-0033, SAND82-1659, p. 272, Oct. 1982.

4.13 Criteria of the ASME Boiler and Pressure Vessel Code for Design by Analysis in Sections III and VIII, Division 2, New York: American Society of Mechanical Engineers, 1969, pp. 6-9.

4.14 Kim, S.B., "Staff Position: Safety Factor for Buckling Analysis of Steel Containment Vessel," Memo to F.P. Schauere, NRC, Oct. 16, 1981.

4.15 ASME Boiler and Pressure Vessel Code, Case N-284, 1980.

5. COMPUTER IMPLEMENTATION

In this section, the procedures outlined in Sections 2 and 3 are implemented to predict the capability of the containment to withstand a nonaxisymmetric local or global dynamic pressure. Following the steps in Sec. 1.3, BOSOR4 (5.1) will be used to determine the mode shapes, natural frequencies and the modal stress resultants of the axisymmetric structure. Secondly, a routine referred to as a LOADS program will be used to evaluate the equivalent axisymmetric point-in-time pressures at specific meridians. These will then be used as an input for BOSOR5 (5.2) to perform the quasi-static analysis discussed in Sec. 3.5.

5.1 B4AMES Analysis of Vibration Modes

A stiffened or smooth shell is idealized for the finite difference analysis using shell segments. A total of 25 segments are allowed in both BOSOR5 and BOSOR4. Section 3.6.1 gives brief guidelines which can be used when preparing the finite difference model. BOSOR4 and BOSOR5 user's manuals also discuss how to model each segment.

Among the several analysis options available in BOSOR4 is a vibration mode analysis of axisymmetric shells. This is referred to as an INDIC=2 analysis. The program calculates the first m meridional modes of vibration for a specified range of circumferential waves, n (see Eq. 3.1). Hence, an INDIC=2 analysis solves Eq. 2.4 to obtain the natural frequencies $[\omega^2]$, mode shapes $[\Phi]$, stress resultants $[N_\Phi]$ and the generalized mass $[M_\Phi]$. Additionally, the modal strains and stresses can be evaluated in an INDIC=2 analysis. Besides being available for printout, the output from BOSOR4 is stored on files created during the run for the upcoming dynamic analysis. These files contain problem title, number of segments used to model the shell, number of mesh points per each segment, geometric parameters (such as curvature, radius, and mesh points spacing), number of modes per each circumferential wave, mode shapes, modal frequencies, modal mass, modal stress resultants and the mass per unit area at each mesh point. The creation of these files has been added to the original BOSOR4 version. The modified version of the BOSOR4 program hereafter is referred to as B4AMES.

5.2 LOADS Program

The purpose of this program is to obtain the equivalent axisymmetric quasi-static pressure for the BOSOR5 analysis (Eq. 3.10) and to calculate the stress intensities to facilitate the ASME Level C Code check. This is accomplished following Sec. 2, Sec. 3, and Sec. 4.2. First the applied nonaxisymmetric pressure $p(s, \theta, t)$ is expressed in a Fourier series (Eq. 2.3). The number of the Fourier terms must correspond to the number of the circumferential waves used in the B4AMES modal analysis (Sec. 5.1). The calculated Fourier amplitudes p_{an}, p_{bn} are then used to evaluate the equivalent nodal force vector $\{F(t)\}$ (Eq. 2.2).

The modal forces are then found by Eq. 2.9. The Wilson - θ method is used to perform the numerical integration to solve for $\{\lambda(t)\}$ from Eq. 2.6. At this stage, all of the modal frequencies are available for the modal analysis (Sec. 5.1) and hence an appropriate time step size can be selected (see Sec. 2.1). The step size is selected by the LOADS routine unless otherwise specified by the user.

Having calculated the modal coordinate $\{\lambda(t)\}$ at each time, the Fourier amplitudes of the displacements and stress resultants are obtained by Eqs. 2.5 and 2.12, respectively. These are used to calculate the displacements, stress resultants, and the stress intensities at any specified meridional location by Eq. 2.1, 2.11 and 4.13, respectively. The equivalent modal axisymmetric pressures at a meridional located at angle θ are obtained by using Eq. 3.9. Finally, the total axisymmetric equivalent pressures at this meridional location are calculated by superimposing the modal pressures using Eq. 3.10. The total pressures are stored in an auxiliary file to be used in the BOSOR5 analysis described in the following section. An auxiliary file will be created for each meridional location specified by the user.

5.3 B5AMES Analysis

The modified version of the BOSOR5 code, i.e., the B5AMES program will be used to perform the bifurcation buckling analysis using an axisymmetric model. The geometric input data for this program is similar to that required for the B4AMES analysis. Material nonlinear behavior can be included in B5AMES by specifying a piecewise linear constitutive relationship. This option is not available in the B4AMES program.

The current BOSOR5 version considers a unique distribution of the pressure-time loading relationship for each segment. In other words, the applied pressure must have the same pressure-time variation for all mesh points within the segment. This distribution is not general enough to handle the present case in which the pressure can vary from point to point. The modified version of the BOSOR5 code allows the user to read the point-by-point pressures for each time from the auxiliary file created during the execution of the LOADS routine.

Failure of the containment to perform its intended function is defined in Sec. 4.1. The resistance of the containment is defined as the pressure p_R which has the same time and space distribution as the input pressure, p, and a magnitude given by:

$$p_R\,(s,\theta,t) = m_p\, p(s,\theta,t) \tag{5.1}$$

In this equation, m_p is a pressure multiplier which is found by the following procedure:

- select a meridian at θ;
- for a selected value of m_p, input the time history of the

equivalent axisymmetric pressures into B5AMES in order to evaluate the shell strains and check stability at all times;

- increase m_p until failure occurs, i.e., strains reach twice yield or bifurcation buckling occurs;
- examine all meridians until the "worst" is found.

5.4 Verification Problems

Before proceeding to the application of the outlined approach to containment analysis, it is appropriate to check and compare results with those from other analyses sources.

5.4.1 Cylindrical Shell Under Uniform Internal Static Pressure

An infinite cylindrical shell with a radius of 200 in. and thickness of 2 in. was subjected to uniform static pressure of 10 psi. The B4AMES modal analysis was performed to find 5 meridional modes for the $n = 0$ circumferential wave. For this static case, Eq. 2.6 can be solved without time integration. Excluding the end points, the circumferential membrane stress resultants range between 1999 to 2027 lb/in. Also, the radial displacement ranged between 0.00691 to 0.00700 in. The exact solution for the circumferential stress resultant and the radial displacement are 2000 lb/in. and 0.00667 in., respectively.

5.4.2 Cylindrical Shell Under Nonaxisymmetric Static Pressure

5.4.2.1 Cosine Load

The cylindrical shell shown in Fig. 5.1 was analyzed by two different approaches. In the first, the INDIC=3 analysis option in B4AMES was used directly. It is specifically for the linear static analysis of axisymmetric shells with nonaxisymmetric loads. The second solution was obtained using the method outlined herein, specialized to the static case. In each of the solutions, the load was expanded in a Fourier series using 0 circumferential waves. The effect of the meridional modes on the accuracy of the second approach was investigated by using 5, 10, and 20 modes, successively. Table 5.1 summarizes the maximum membrane circumferential and meridional stress resultants and radial displacements at two different meridional locations ($\theta = 0$ and 90 degrees) by both analysis methods. Additionally, the distribution of the stress resultants along the $\theta = 0$ meridian is shown in Figs. 5.2 and 5.3. From the table and the figure, one can see that the results of INDIC=3 analysis agree with those obtained with the LOADS program when twenty meridional vibration modes are employed.

5.4.2.2 Three Point Loads

An infinite cylindrical shell subjected to a static pressure with the distribution shown in Fig. 5.4 was analyzed. Again, both an INDIC=3 analysis and a modal solution, as outlined herein, was performed. In the second solution, 5, 10, and 20 modes were used for each circumferential wave number n. Twenty circumferential Fourier terms were used from 0 to 57 in increments of 3, i.e. $n = 0, 3, 6, \ldots 57$. Other Fourier terms will be zero because of the circumferential load distribution. The shell was modeled for the finite difference analysis using 18 mesh points. Symmetry conditions were imposed midway between the support. Table 5.2 illustrates the difference between the results of the two approaches. The table lists the maximum membrane stress resultants and radial displacement at three different meridional locations (θ = 0, 5 and 60 degrees). The solution using 5, 10, or 20 vibration modes for each circumferential wave is in close agreement with INDIC=3 analysis option.

5.4.3 Time Response of a Cylindrical Tube

A simply supported cylindrical tube (Fig. 5.5(a)) subjected to a suddenly applied circumferential load (Fig. 5.5(b)) was analyzed. The tube was idealized using one shell segment divided into 17 elements with a size equal $\sqrt{rt}/2$. The symmetry conditions midway between the support were imposed on the model. Ten and 20 meridional modes and an $n = 0$ circumferential wave were used. From the modal vibration analysis using 10 and 20 modes, the smallest natural periods were found as 0.000041 and 0.000019 seconds, respectively. However, a time step size equal to that in Ref. 5.3 was employed (Δt = 0.00001 sec.). The time response of point "A" using the LOADS program is shown in Fig. 5.5(c). Superimposed are the theoretical and the finite element results from SAPIV given in Ref. 5.3. The figure illustrates good agreement between the theoretical results and the results of the LOADS program using 20 meridional modes. Ten vibration modes underestimate the time response of the tube. The peak response calculated using 20 and 10 modes is 11.73 (10^{-4}) in. and 11.13 (10^{-4}) in., respectively.

Additional studies were carried out to investigate the effects of the mesh size. The analysis was again performed twice using 10 and 20 modes but with an element size of $\sqrt{rt}/4$. The peak response was predicted as 11.0 (10^{-4})in. and 11.36 (10^{-4})in., respectively.

5.4.4 Summary

In summary, the modal analysis technique in conjunction with the LOADS routine can be used to solve nonaxisymmetrically loaded shells. The number of the meridional and circumferential modes of vibration affects the accuracy of the results. A larger number of meridional modes is required in cases where the applied loads vary rapidly along the shell meridional direction and fewer modes are necessary to analyze shells with loads which vary gradually in the meridional direction (see Secs. 5.4.1, 5.4.2.1, 5.4.3, and Table 5.1). In the same way, enough circumferential waves must be used when the applied pressure varies rapidly in the circumferential direction (see Sec. 5.4.2.2 and Table 5.2).

5.5 Analysis of a Containment Vessel

To demonstrate the application of the procedure outlined in this report to containment analysis, a simplified containment was analyzed. The vessel consists of a circumferential stiffened cylindrical shell covered with a 2:1 ellipsoidial head. Fig. 5.6 illustrates a schematic of the containment. The siffeners have rectangular cross sections of 36 in^2. Elastic-perfectly plastic material properties with a yield strength and modulus of elasticity of 50 ksi and 29,000 ksi, respectively, are used.

To describe the pressure transients applied to the vessel, the wall is subdivided into five regions (Fig. 5.7). In real containments, the number of compartments is several times the five regions used herein. However, this will be sufficient for demonstration purposes. Fig. 5.7(a) illustrates the pressure versus time transients in each region and Fig. 5.7(b) shows the circumferential variation of the pressure at 0.3 seconds. Notice that the maximum pressure is one psi. Hence, m_p in Eq. 5.1 represents the maximum value of the pressure in Region Two.

The analysis of this containment was accomplished following Secs. 5.1, 5.2 and 5.3. The vessel was modeled using three shell segments with a total number of 159 mesh points. The finite difference element length was limited to $\sqrt{rt}/2$. A fixed support condition was assumed at the containment base. No imperfection was included in the BOSOR4 model since the vibration frequencies and modes are relatively insensitive to imperfection (5.1, pp. 34). The input data for the interactive VAX version of B4AMES for the modal vibration analysis is given in Table A.12.

The next step is to evaluate the time varying equivalent axisymmetric pressures to be employed in the B5AMES analysis. This step will be accomplished using the LOADS program (see Sec. 5.2). The input procedures are described in the Users's Manual (see Appendix A, Sec. A.3.2). Sec. A.3.3 also lists the data which describes the distribution of the applied pressure shown in Fig. 5.8. The nonaxisymmetric pressure was expanded using 11 Fourier terms (n = 0, 1, 2, 3, ... 10).

As discussed with the verification problems, the number of meridional modes, m, has an important effect on the accuracy of the final results. A study was conducted using n = 0, 1, 2 and 3 circumferential waves results. (These are the most dominate terms in the Fourier expansion of the pressure shown in Fig. 5.7). The changes in the stress resultants over the three shell segments were examined as m was increased. Figure 5.8, which illustrates the variation in the maximum circumferential stress resultant in Segment Two at 0.03 seconds versus the number of the meridional mode m, is typical. Twenty meridional modes for all circumferential wave numbers greater than zero is adequate. On the other hand, this number was found to be insufficient to represent the axisymmetric portion of the dynamic behavior, particularly between the ring stiffeners near the base. Forty meridional modes will be used for the n equal zero circumferential wave.

From the modal analysis using $m = 40$ for $n = 0$ and $m = 20$ for $n = 1, 2, \ldots 10$, a time step size of 0.001634 seconds based on Sec. 2.2 would be adequate. However, to obtain output at more convenient times, a value of 0.0015 seconds was employed. Eq. 2.6 was numerically integrated from zero to one second. No damping was included in the analysis. The equivalent axisymmetric pressures are calculated every 0.03 seconds at four different meridional locations: $\theta = 0, 45, 90, 180$ degrees.

Following Sec. 5.3, a finite difference B5AMES model of the containment was established. This model is the same as that used in the modal vibration analysis but with a geometric imperfection included as per Sec. 3.3.3. The imperfection is taken as given in Eq. 3.3. The imperfection amplitude was calculated as 1.5 in. using Eq. 3.4 where the out of roundness coefficient, γ, (Eq. 3.5) was taken equal to one-half. The ASME tolerance on this value is one (5.4) so that a mean value of one-half is reasonable. Additionally, the imperfection wave length was chosen to satisfy the requirements in Sec. 3.3.3. Table A.15 in Appendix A summarizes the input data for the VAX version of B5AMES.

The B5AMES analysis was accomplished by first using the equivalent axisymmetric pressure at $\theta = 0$ as an input. The solution was repeated using the INDIC=0 option (nonlinear static analysis) while increasing the pressure multiplier, m_p, (Eq. 5.1). In each case, the membrane strains were examined. The maximum value was in the circumferential direction and occurred midway between the first two rings near the containment base. Fig. 5.9 illustrates the relationship between this maximum membrane strain and the pressure multiplier. From the figure, the maximum membrane strain reached twice the yield value with a pressure multiplier of 110 (Point C in Fig. 3.1). This represents a peak dynamic pressure of 110 psi in Region Two.

The meridional locations at $\theta = 45, 90,$ and 180 were next investigated. At a pressure multiplier equal to 110, all the membrane strains on these meridians were less than those found for $\theta = 0$. In other words, the $\theta = 0$ meridional location is the "worst" meridian.

The final step in the containment analysis is to investigate the stability. This was performed using the INDIC=-3 analysis option in the B5AMES code. For circumferential wave numbers ranging from 0 to 50 and a pressure multiplier of 110, a bifurcation point did not occur, i.e., Point B is beyond Point A in Fig. 3.1. Therefore, the failure pressure of the containment shown in Fig. 5.6 subjected to the pressure distribution given in Fig. 5.8 is 110 psi and controlled by plastic collapse. Failure occurs at 0.3 seconds, i.e., when the peak pressure is reached. Because of the relatively high shell frequencies, the response is probably quite close to the static response, though this was not investigated. The normal displacement of the containment at

0.3 seconds is shown in Fig. 5.10. This was obtained from the LOADS program results using Eq. 2.5.

5.6 ASME Level C Code Check

The ASME Level C Code criteria summarized in Sec. 4.2 was also checked. The membrane stress intensities at each mesh point along the four meridional location (θ = 0, 45, 90 and 180) were calculated using Eq. 4.13. The envelopes of these stresses were constructed by the LOADS program following Sec. A.4. These envelopes correspond to a peak pressure of 1 psi. Figure 5.11 illustrates the stress intensity envelope at these four meridional locations. The maximum stress intensity value of 682.5 psi occurred at $\theta = 45$ degrees and between the first two rings near the base of the containment. At a distance of $\sqrt{rt}$ away from the peak stress, the stress intensity is 630 psi. Therefore, the controlling stress is classed as general primary membrane stress (Eq. 4.10). Proportioning the maximum stress intensity up to the yield stress value of the vessel material (S_y = 50 ksi) yields a peak pressure multiplier of 73.25 psi. No pressure corresponding to ASME Level C buckling criteria is selected for this containment. However, the BOSOR5 buckling analysis in the previous section can be assumed to fulfill the requirement of the rigorous analysis described for S_{cr} in Sec. 4.2.

5.7 References

5.1 Bushnell, D., "BOSOR4 Program for Stress, Buckling and Vibration of Complex shells of Revolution," Structural Mechanics Laboratory, Lockheed Missiles and Space Co., Inc., Palo Alto, CA, 1975.

5.2 Bushnell, D., "BOSOR5 Program for Buckling of Elastic-Plastic Complex Shells of Revolution Including Large Deflections and Creep," Structural Mechanics Laboratory, Lockheed Missiles and Space Co., Inc., Palo Alto, CA, 1975.

5.3 Bathe, Klaus-Jurgen, Wilson, E.L. and Peterson, F.E., "SAPIV: A Structural Analysis Program for Static and Dynamic Response of Linear Systems," EERC 73-11, Earthquake Engineering Research Institute, Berkeley, 1973.

6. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

6.1 Summary

The function of the containment structure is to restrict leakage for a variety of environmental and accident loading conditions. Although very rare, loadings more severe than the design loading can be postulated. Besides uniform internal pressure, severe accident loadings on containments include dynamic and seismic loadings. The objective of this work is to develop a methodology for assessing the ultimate capacity of steel containments due to global or local, uniform or spatially varying dynamic loadings. The scope of this work is limited to unpenetrated steel containments. Failure modes of penetrations, anchorages and any other modes are not considered. Vessel resistance is predicted based upon either a nonlinear large displacement analysis or the ASME Boiler and Pressure Vessel Code Level C criteria.

The containment vessel experiences various types of loadings that vary in time and both circumferential and meridional directions. A three-dimensional finite element model could be used to analyze these variations but, in most cases, would be impractical. In this work an axisymmetric geometric model is used and the circumferential variation of the pressure is represented by a Fourier Series. The response of the containment is expressed in terms of the modal coordinates. The Wilson θ method is used to solve numerically the modal equilibrium equation. The displacements and the stress resultants at any point of time are represented by Fourier series at any point of time. Values of these components at discrete locations are evaluated using modal superposition.

Plastic collapse of the containment shell can occur in either tension or compression regions whereas bifurcation buckling is associated solely with compressive stresses, e.g., in the lower portion of the cylindrical shell during seismic loading. Consistent with the use of an axisymmetric containment model, an axisymmetric "worst meridian" prebuckled shape is used to investigate buckling. The effects of the circumferential variation of the stress resultants on the shell buckling strength are studied. All of the investigations conducted regarding this variation have concluded that the buckling stress is not sensitive to this variation if the variation is slow with respect to the buckled shape. The "worst meridian" approach is found to yield to a conservative value of the shell buckling strength. Shell vibration and buckling analyses are performed using modified BOSOR4 and BOSOR5 finite difference codes, respectively. These programs were chosen over the other programs since they are available to the NRC staff on the Brookhaven National Laboratory computer facilities. BOSOR5 results are calibrated with experimental results. Geometric imperfections are modeled in the meridional direction by a cosine deviation from the perfect shape. No imperfections in the circumferential direction were included. Other nonaxisymmetric factors such as penetrations cannot be modeled in the BOSOR5 axisymmetric model. However, several experimental investigations are cited to show that penetrations with diameters less than $2\sqrt{rt}$ and reinforced in accordance with the ASME area replacement rules do not affect the shell buckling load. A quasi-static solution summing the applied dynamic and inertial pressures is used. These pressures' input into the BOSOR5 model are equivalent pressures that produce the correct stress resultants along the "worst" meridian under consideration. Some changes are introduced into the current BOSOR5 version to accommodate point wise variation of these pressures. The new versions of BOSOR4 and BOSOR5 programs are referred to as B4AMES and B5AMES, respectively.

Failure of the containment (leakage) is defined to occur when the maximum membrane strain in the shell reaches twice the yield strain. This strain will be reached by loading along a fundamental loading path unless compressive stresses introduce a bifurcation point. It is assumed that, if a bifurcation point is reached, buckling will occur along a secondary path and the strain will exceed twice the yield value along this path. The ASME Boiler and Pressure Vessel Codes Level C criteria is also applied as a failure condition. The ASME membrane stress intensity is calculated at each mesh point for specified meridians at each time. The pressure peak which produces the ASME Level C allowable stress intensity is determined.

The analysis methodology was implemented into a computer code called LOADS. This program evaluates the equivalent axisymmetric point-in-time pressures and the point-in-time stress intensities at specific meridians using B4AMES modal analysis output and the nonaxisymmetric applied dynamic pressure. The equivalent pressures are used as an input to the B5AMES analysis. Several verification problems were analyzed to check the results of this methodology with those from other analyses. The number of the meridional and circumferential modes of vibration in the analysis has a significant effect on the accuracy of the results. A simplified containment vessel was also analyzed to demonstrate the application of the outlined methodology.

A User's Manual for the LOADS program and the modified BOSOR programs is presented. Samples of input and output as well as the files that will be created during the execution of the B4AMES, LOADS and B5AMES codes are discussed. The attached sample input data are typical of those required to analyze a containment structure.

6.2 Conclusions

The approach outlined in this work can be used to assess the ultimate capacity of steel containments under global or local dynamic loads. The methodology could assist the Nuclear Regulatory Commission in their regulatory and licensing activities regarding severe accident loadings. The computer program written to apply this approach to a containment vessel is easy to follow. The user can evaluate containment vessel resistance based on either plastic collapse bifurcation buckling or Level C Code criteria.

6.3 Recommendations

Perhaps the biggest shortcoming of the analysis procedure is that the user must prepare two separate input files for the B4AMES and B5AMES programs. Future work should be devoted to writing one preprocessor that can be used for both programs. Also, nonlinear dynamic behavior should be incorporated. This requires significant effort but some analysis techniques are available. Additionally, of course, there is always the problem of the buckling of thin shells that could use more study. The containment vessel strength under combined seismic and pressure loadings should also be examined. Further studies should investigate the relationship of buckling to leakage.

A. APPENDIX A - USER'S MANUAL

This User's Manual describes the computer program (LOADS) that was developed and used in conjunction with the modified versions of BOSOR4 and BOSOR5 to analyze shell structures under global or local dynamic pressure loadings. The modified BOSOR4 and BOSOR5 programs are referred to here as B4AMES and B5AMES, respectively. The following describes the procedure as well as the VAX commands that can be used to execute the B4AMES, LOADS and B5AMES programs on the Brookhaven National Laboratory (BNL) VAX 11/780 system. The authors strongly recommend that one should know how to use the original BOSOR4 and BOSOR5 programs before using the modified versions of these programs. If the B4AMES, LOADS and B5AMES are stored on a tape, the user should load them on the VAX system following the steps oulined in Sec. A.1. If these programs are on the VAX, the user may skip to the steps given in Sec. A.2 which can be used to execute the B4AMES, LOADS and B5AMES programs. If the command files in Sec. A.2 are available, the user may skip to Sec. A.3 to execute the programs.

A.1 Loading the B4AMES, LOADS and B5AMES Codes on the BNL VAX System

Let USER1 be the user name where these programs are to be stored on the VAX. Loading these three programs and the associated command files on the VAX system can be accomplished as follows.

- Log in using USER1 name.
- Create the log in file shown in Table A.1.
- Create three subdirectories by typing:

  ```
  $ CRE/DIR [.B4AMES]

  $ CRE/DIR [.LOADS]

  $ CRE/DIR [.B5AMES]
  ```

- Log out.
- Log in using USER1.
- Load the B4AMES, LOADS, B5AMES and the associated VAX command files on the VAX using the following commands:

  ```
  $ ALLOCATE MTAO:

  $ MOUNT/FOR/BLOCKSIZE=16004 MTAO: NRCAL/COMMENT
    ="LOAD TAPE N27993"

  $ BACKUP MTAO:B4B5/SEL=[POLK.B4AMES]*.*.*
    [USER1.B4AMES]*.*.*
  ```

```
$ BACKUP MTA0:B4B5/SEL=[POLK.LOADS]*.*.*
  [USER1.LOADS]*.*.*

$ BACKUP MTA0:B4B5/SEL=[POLK.B5AMES]*.*.*
  [USER1.B5AMES]*.*.*
```

where N27993 is the tape number.

The last three commands are used to copy the B4AMES, LOADS and B5AMES programs to the VAX in subdirectories having the same name. After a successful completion of this copy procedure, the user can then start using these programs following Sec. A.2.

A.2 Command Files

The following represents the sequence of the VAX commands required to execute the B4AMES, LOADS and B5AMES program.

B4AMES Program:

Command Name	**Command Description**
B4CON	To assign some of the B4AMES files to Units 2, 4 and 7 used in providing the input data interactively (Table A.2).
B4PRS	Runs the B4AMES preprocessor to create the required BOSOR.DAT file.
B4PRA	Assembles the provided data in one file called BOSOR.DAT (Table A.3).
B4MI or B4MS	Initiates an interactive or a batch run, respectively, of the B4AMES main processor (Table A.4).
B4PLO	Initiates an interactive plotting of the B4AMES program results (Table A.5).

LOADS Program:

Command Name	**Command Description**
LDMI or LDMS	Initiates an interactive or a batch run of the LOADS program (Table A.6).

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B5AMES Program:

Command Name	**Command Description**
B5CON, B5PRS*, B5PRA*	Used for the same purpose as in the B4AMES program.
B5PRE	Execute the B5AMES preprocessor to create the required BOSOR.DAT file.
B5MAS	Initiates an interactive run used to provide the main processor data. (Table A.7)
B5MI, B5MS	As explained in the B4AMES Program.* (Table A.8)
B5POS	Initiates an interactive run used to provide the postprocessor data. (Table A.9)
B5POI, B5POQ	Execute an interactive or batchrun of the B5AMES program postprocessor. (Tables A.10, A-11)
B5PLO	As in the B4AMES program.

*This command file is the same as this used with the B4AMES program with the number 5 replacing 4.

A.3 B4AMES Program

A.3.1 B4CON

The user must issue this command when starting the analysis of a new problem by typing B4CON. This command assigns the prompt data files to Units 2, 4 and 7 (see Table A.2) which are required to run the B4AMES preprocesser interactively.

A.3.2 B4PRS

In this step the user executes the B4AMES preprocessor by typing B4PRS. As explained in Ref. (A.1), the data is provided on a segment by segment basis in a conversational mode, i.e., the program prompts the user with the required input. Additionally, the user can get more information regarding the input by typing Help. At the end of providing the data, an N+1 file called FOR003.DAT is created, where N is the number of segments. The N + 1th file contains the boundary conditions and information about the loading variation with respect to time. The user can edit any of these files to change any of the provided data. However, the file version number must not change.

A.3.3 B4PRA

After being satisfied with the provided data, the user issues a command which assembles the above N+1 FOR003.DAT files into the B4AMES program input deck called BOSOR.DAT. This is accomplished by typing B4PRA (see Table A.3). The BOSOR.DAT file contains the data ordered in the sequence given in the BOSOR4 program user's manual. It is noteworthy to mention here that the N+1 FOR003.DAT files are automatically deleted after a complete assembly. Therefore, it is recommended to copy these N+1 FOR003.DAT files to other temporary file for future use. Table A.12 is a list of the BOSOR.DAT file used in the modal vibration (INDIC=2) analysis of the vessel shown in Figure 5.6. This input was generated following BOSOR4 User's Manual (A.1) and the guidelines discussed in Sec. 3.6.1.

A.3.4 B4MI and B4MS

Having created the BOSOR.DAT file, the user issues a command which can initiate either an interactive or batch run of three processors: preprocessor, mainprocessor, and postprocessor. In the B4AMES program, these are executed simultaneously. The interactive run is accomplished by typing B4MI while the command B4MS initiates a batch run of the B4AMES program. The problem solution is stored in files named B4AMES.OUT, FOR040.DAT and FOR053.DAT in the subdirectory used to execute the B4AMES program. The data stored in files FOR040.DAT and FOR053.DAT is needed for plotting the results of the analysis. These two files are created if the plot option NLAST (see Table A.3) is zero or one. The information listed in B4AMES.OUT is controlled by the user-specified printout option, NPRINT. If detailed information regarding, for example, the input, execution time, and block size is desired, NPRINT should be specified as 3, i.e., maximum printout. In the problem investigated herein, a medium printout was specified (NPRINT=2). In this case, B4AMES.OUT file will contain containment geometric parameters, mode shapes, modal frequencies and modal stress resultants. This information is also stored on a file called FOR014.DAT since it is needed in the next step to perform the shell dynamic analysis. To accomplish this, some FORTRAN statements were added to the original file in BOSOR4 to write the following on the FOR014.DAT File:

Original File Name of BOSOR 4	**File Name Used in B4AMES**	**Items Written on FOR014.DAT**
READIT	REAMES	Problem title, number of segments, starting, ending and increment number of circumferential waves.
GEOM	GEOMAMES	Number of mesh points per segment and segment geometric parameters.

MODE	MODEAMES	Modal stress resultants, frequencies, mode shapes, and geometric parameters needed to calculate the nodal force vector.

With these modifications, the program is referred to as B4AMES. The FOR014.DAT file is stored in the subdirectory names B4AMES. It is noteworthy to mention here that when executing the B4AMES program, an echo copy of the BOSOR.DAT with updated annotations is created. This is to be used to check the input data when any error in the input data is encountered during the run.

A.3.5 B4PLO

Having successfully executed the B4AMES program, the user issues a command which initiates an interactive plotting sequence of the analysis results. These plots represent either the geometry or the geometry and the displacement profile along the shell wall. The command file listed in Table A.5 can be used for this purpose and is accomplished by typing B4PLO.

A.4 LOADS Program

A.4.1 Outline of the Computer Code

This computer program was developed to interface B4AMES and B5AMES codes for the containment dynamic analysis. The variation of the pressure with respect to time, meridional and circumferential locations are specified in this program. They are expanded into a Fourier series for the specified circumferential wave numbers (Eq. 2.3). For each Fourier component the load vector is formed and the characteristic equation (Eq. 2.6) is solved in the time domain by using the Wilson - θ method. After solving for the modal coordinates, the contribution of all modes is summed up to obtain the total response and the equivalent axisymmetric pressures at specified meridians using Eqs. 2.5, 2.12, and 3.10. The solution vectors $\{\lambda\}$, $\{\dot{\lambda}\}$ and $\{\ddot{\lambda}\}$ at the final solution step are stored on a file for a restart possibility. The vectors $\{\lambda(0)\}$ and $\{\dot{\lambda}(0)\}$ are automatically set equal to zero when analyzing a new problem.

The LOADS program calculates the membrane stress intensity at each mesh point for specified meridians at each time (Eq. 4.13). To facilitate a ASME Level C analysis (Sec. 4.2), the stress intensities are enveloped in time and θ. The resulting output provides the maximum stress intensity for each meridional mesh point s for all θ and all t. The θ and t corresponding to the maximum stress intensity at each s is retained.

A.4.2 Input Data

All input units must be consistent. Recommended input units are lbs., in., sec. and degrees. The input data on each line is separated using either commas or blank spaces. The user must create an input data file called LOADS.DAT with the following information.

READ NMERD, NPRESS, NPSTRE, NASME, NPRINT, NSTRT, NDTOP, DAMRAT, ROW

NMERD = Number of circumferential locations to be investigated to determine the "worst meridional" location.

NPRESS = Number of time increments Δt's between the calculation of the equivalent axisymmetric pressures.

NPSTRE = Number of time increments Δt's between the calculation and printout of the stress resultants and displacements.

NASME = Number of time increments Δt's between the updating of the stress intensity envelope.

NPRINT = Printout option.
0 = minimum printout
1 = maximum printout

NSTRT = Restart option.
0 = new problem
1 = to continue previously analyzed problem

For a restart, a file called FOR030.DAT must be available. (See Sec. A.4.3.)

NDTOP = Numerical integration option.
0 = Δt and the total integration steps are determined internally by the program.
1 = The user must specify Δt and the total number of the integration points.

DAMRAT = Damping ratio. This is assumed constant for all modes of vibration.

ROW = Shell skin material mass density.

READ (THETA(II), II = 1,NMERD)

THETA = Meridional locations to be investigated to check the "worst" location.

READ DT, NT1 if NDTOP is specified one on the first line.

DT = Time Step Size.

NT1 = Total number of integration points.

READ NSEGL, NTT

NSEGL = Number of loaded segments. This is not necessarily equal to the number of segments used to model the structure in the BOSOR4 analysis.

NTT = Total number of points in time required to define the pressure-time history (maximum of 100 points).

DO 5 JT = 1,NTT

READ T (JT)

T(JT) = Current time on the pressure-time function where the circumferential variations of the pressure are to be specified.

DO 10 I = 1, NSEGL

READ ISEGL, NX(I)

ISEGL = Current segment number (1 to NSEGL).

NX(I) = Number of circumferential stations used to define the circumferential load variation. (100 maximum)

DO 20 I1 = 1,NX(I)

20 READ XPLUS(I1), YPLUS(I1), XMINUS(I1), YMINUS(I1)

XPLUS(I1) = positive circumferential coordinate in degrees.

YPLUS(I1) = pressure amplitude at XPLUS.

XMINUS(I1) = negative circumferential coordinate in degrees.

YMINUS(I1) = pressure amplitude at XMINUS.

Notice that the above input follows that used in the BOSOR4 INDIC=3 analysis.

READ J1, PJ1 (One line is required for each mesh point in the current segment to define the meridional pressure variation.)

J1 = mesh point number.

PJ1 = normal pressure at mesh point J1.

Meridional pressure lines must be in numerical sequence. If lines are omitted, the missing mesh point's pressure is generated in the LOADS program using linear interpolation based upon the mesh points spacing. The pressures at the first and last mesh points in ISEGL must be provided by the user, otherwise, the execution will be terminated and an error message will be printed.

```
10 CONTINUE
 5 CONTINUE
```

Note that, for example, the pressure on the XPLUS meridian at mesh point J1 is equal to YPLUS times PJ1. Hence, when specifying the variation of pressure $p(s,\theta,t)$, it is convenient to normalize either the circumferential variation (YPLUS and YMINUS) or the meridional variation PJ1.

The source code of the LOADS computer program is listed in Table A.13 followed by the input data, Table A.14, used to analyze the vessel shown in Figure 5.6. The LOADS routine is written using the FORTRAN language. The enclosed list of this program is for the VAX 11/780 minicomputer. The program is limited to 250 longitudinal modes, 500 mesh points over the entire structure and 50 Fourier terms. some comments have been added to describe the listed data. The user should not include any of these comments when executing the LOADS program.

A.4.3 Results and Output of the LOADS Program

After being satisfied that the data in LOADS.DAT is correct, the user issues a command to batch or run interactively the LOADS program. The interactive run is accomplished by typing LDMI while the command LDMS can be used for the batch job (see Table A.6). It is recommended to use the batch alternative to execute a job that requires long CPU time.

During the execution of the LOADS program, several files with the following information are created and stored in the subdirectory LOADS.

LOADS.OUT

This file contains an echo of the input data printed out in formatted statements. A coding for each printout is given to enable the user to follow and check the provided input. The calculated Fourier amplitudes for each of the circumferential wave numbers are listed. Additionally, the input data from the modal vibration analysis can be listed if a value of one is specified for the printout option (NPRINT). The time step size and the number of steps used to perform the solution of Eq. 2.6 are printed. Stress resultants and displacements at the different meridional locations are listed every NPSTRE time steps. The envelopes of the stress intensities for all mesh points at all meridional locations are listed. Also, printed out is the envelope of these envelopes. Finally, the maximum value of the stress intensities among all envelopes is printed out along with its location and time of occurrence.

PRESSR.DAT

This file contains the equivalent axisymmetric pressures to be used in B5AMES analysis. Several files called PRESSR.DAT;1, PRESSR.DAT; 2, PRESSR.DAT;3, ... PRESSR.DAT, NMERD are created by the LOADS program when starting the analysis of a new problem (NSTRT is zero). One file is used for each meridional location to store the equivalent axisymmetric pressures calculated every NPRESS. These files are extended to store additional equivalent axisymmetric pressures if the restart option NSTRT is defined as one.

FOR030.DAT

This file contains the solution of Eq. 2.6 at the end of the previous analysis step. In other words, the vectors $\{\lambda\}$, $\{\dot{\lambda}\}$ and $\{\ddot{\lambda}\}$ evaluated at the time T(NTT) are stored on this file for a possible restart. The current envelope of the stress intensities is also stored. This file must be available when NSTRT is specified on the control information line as one. The total number of FOR030.DAT versions is equal to the number of the restart plus one.

A.5 B5AMES Program

The B5AMES program is used to predict the containment vessel resistance. The solution is accomplished in two steps. In the first, the pressure which produces membrane strains equal to twice the yield strain (Sec. 4.1) is predicted. The nonlinear axisymmetric analysis option in B5AMES program (INDIC=0) is used. The second step is to investigate stability using the INDIC=-3 analysis option.

Unlike B4AMES, the B5AMES preprocessor, mainprocessor and postprocessor are run individually. Each of these processors is run in two steps: (1) creation of the processor data and (2) execution of the processor.

A.5.1 B5AMES Preprocessor

A.5.1.1 Preprocessor Input (INDIC=0, -3) - B5CON, B5PRS and B5PRA

The steps to be followed to create a BOSOR.DAT file are those used to create the B4AMES program data file. In other words, the user follows Sections A.3.1, A.3.2, and A.3.3 to provide the data required to execute the B5AMES program. Table A.15 lists the BOSOR.DAT file used in the analysis of the containment vessel shown in Figure 5.6. This was generated following the BOSOR5 User's Manual (A.2) and the guidelines discussed in Sec. 3.6.2. Notice that the pressure keys PN and PT in all segments are defined as zero. This is because the equivalent axisymmetric tangential and normal pressures will be provided from the external files created by the LOADS program (PRESSR.DAT, see Sec. A.4.3). To accomplish this, some changes were added to the original BOSOR5 program. These changes were added to the files calles PRE11 and MAIN used in the original BOSOR5 version. The updated files used in the B5AMES program is called PRAMES11 and MAINAMES, respectively.

A.5.1.2 Execution of B5AMES Preprocessor - B5PRE

The command listed in Table A.7 if for an interactive execution of the B5AMES preprocessor. It is assumed that all the data is available on the BOSOR.DAT file. The execution is accomplished by typing B5PRE. After a successful execution of this process, three files called FOR012.DAT, FOR011.DAT and BOSORREAD.OUT are created. The first two files are to be used in the execution of the B5AMES main processor. The BOSORREAD.OUT is an echo of the input data and can be used for the check of the provided data. On the other hand, if the execution of the preprocessor fails to be completed, only one file referred to as BOSORERR.DAT will be created. This file contains the data up to the point where the error was encountered. The user should use this file to locate the error and correct that on the BOSOR.DAT file. This step is to be repeated until a successful execution of the preprocessor is completed.

A.5.2 B5AMES Analysis (Mainprocessor)

A.5.2.1 Mainprocessor Input (INDIC=0) - B5MAS

The user must create a file which contains the analysis control options. This is accomplished interactively by typing B5MAS. The provided data is stored on a file called FOR010.DAT which must be available for the execution of the main processor. Table A.16 and A.17 are typical files required to start and restart the B5AMES analysis. The last two lines which must be inserted at the end of the input data FOR010.DAT are, respectively, a control option (LAMES) and the pressure multiplier m_p (see Eq. 5.1) which are shown as 1 and 100, respectively, in Tables A.16 and A.17. The control option is defined as:

- LAMES > 0 The applied pressures will be provided from an external file created by the LOADS program, i.e., the PRESSR.DAT file must be available.

- LAMES = 0 The applied pressure as specified in the BOSOR5 preprocessor will be used.

These can be inserted using the VAC editor. The other control information listed in the above file is described in the BOSOR5 User's Manual (A.2).

A.5.2.2 Execution of B5AMES Mainprocessor - B5MI and B5MS

Having created the mainprocessor data, i.e., file FOR010.DAT, the user can issue a command which can initiate either an interactive (B5MI) or batch (B5MS) run of the B5AMES mainprocessor. The user should specify which PRESSR.DAT file is to be used in the analysis. This can be accomplished by specifying the version number (the "*" in the "PRESSR.DAT;*") which indicates the meridional location under consideration (see Table A.8). The results from execution of the B5AMES mainprocessor are stored in a file called BOSORMAIN.OUT. This file contains the pressures, maximum displacement and the location and magnitude of all plastic strains. However, no information is provided to determine where these strains occur within the shell thickness. More results can be found by running the B5AMES postprocessor.

A.5.3 B5AMES Output (Postprocessor)

A.5.3.1 Postprocessor Data (INDIC=0) - B5POS

As with the B5AMES mainprocessor, the user must create a file containing output control options before executing the postprocessor. This is accomplished by typing RUN POSTSETUP which initiates an interactive run of the BOSOR5 postprocessor. In the VAX version, this is provided interactively and the response is stored in a file referred to as FOR010.DAT. The file given in Table A.18 represents a list of the typical FOR010.DAT file used. Notice that the user can control the printout for each segment. In other words, the user has the option to print the results for a specific segment, specific number of mesh points and the stresses and strains through the shell thickness. In the B5AMES postprocessor data file, the control options LAMES and the pressure multiplier m_p are not required to be defined.

A.5.3.2 Execution of B5AMES Postprocessor - B5POI, B5POQ

Having created the above file, the user issues a command to execute the B5AMES postprocessor. This can be accomplished interactively by typing B5POI or as a batch job by typing B5POQ. The output file is called BOSORPOST.OUT. The last step in this INDIC=0 analysis is to check through this file to determine if twice the yield strain has been reached by a membrane strain. If not, the user should start the analysis over following Sec. A.5.2 with an increased value of m_p.

A.5.4 B5PLOT

This command is used to get plots of the B5AMES analysis results. The procedure to be followed is the same as explained in Sec. A.3.5.

A.6 Stability Analysis (INDIC=-3)

The stability of the containment is investigated using the INDIC=-3 analysis option. In this analysis, the stability determinant for a specified range of circumferential waves is calculated for a sequence of time steps. The results from a previous INDIC=0 (Axisymmetric) analysis must be available. The bifurcation point in an INDIC=-3 analysis is defined to occur when the stability determinant changes sign for any circumferential wave number. The user should check stability for all load steps in the nonlinear static analysis (INDIC=0). The B5AMES mainprocessor file needed to perform the INDIC=-3 analysis can be created following Sec. A.5.2. Table A.19 is a typical FOR010.DAT file required to perform the stability analysis. Once again notice that the last two lines are to be added by the user.

The output of this analysis is stored in a file called BOSORMAIN.OUT which contains the value of the stability determinant for every circumferential wave number and every load step. The INDIC=-3 analysis will be terminated if the stability determinant changes sign. If not, a message is printed at the end of the BOSORMAIN.OUT file indicating that no instability occurred for the prescribed circumferential wave numbers and load steps.

A.7 References

A.1 Bushnell, D., "BOSOR4 Program for Stress, Buckling and Vibration of Complex shells of Revolution," Structural Mechanics Laboratory, Lockheed Missiles and Space Co., Inc., Palo Alto, CA, 1975.

A.2 Bushnell, D., "BOSOR5 Program for Buckling of Elastic-Plastic Complex Shells of Revolution Including Large Deflections and Creep," Structural Mechanics Laboratory, Lockheed Missiles and Space Co., Inc., Palo Alto, CA, 1975.

Table 3.1. BOSOR5 Comparison to Experimental Results

Shell Type	Load Type	Number	Experiment/BOSOR5 Result Mean	C.O.V.	Reference
Torispherical Head	Internal Pressure	8	1.06	0.082	[3.29]
Torispherical Head	Internal Pressure	6	0.97	0.120	[3.30]
Ellipsoidal Head	Internal Pressure	5	1.09*	0.209	[3.28]
Ellipsoidal Head	Internal Pressure	10	1.09*	0.061	[3.28]
Stiffened Cylinder	External Pressure	69	0.960	0.057	[3.4]
Cylinder/Cone	External Pressure	3	1.008	0.007	[3.3]
Torisphere/Cylinder	External Pressure	4	1.021	0.034	[3.3]
Cylinder	Axial Compression	10	1.019	0.058	[3.3]
Cylinder	Axial Compression	24	0.968	0.089	[3.3]
Stiffened & Unstiffened	Axial Compression	40	0.995	0.117	[3.1, 3.3]
Stiffened Cylinder	Axial Compression	5	1.459	0.284	[3.5, No. 5]
		Weighted	1.00	0.135	

---

*Experimental pressures reduced by 20 percent as suggested in [3.26].

Table 5.1 Maximum Membrane Stress Resultants and Radial Displacement for the Cosine Load Problem (Fig. 5.1)

Meridional Location (degrees)	Stress Resultants	LOADS Program			INDIC=3 Analysis
		m = 5	m = 10	m = 20	
0.0	N_1 (lb/in.)	56	84.5	92.9	94.75
	N_2 (lb/in.)	78.4	105.8	125.5	124.6
	w (in.)	.00139	.00156	.00159	.0016
90.0	N_1 (lb/in.)	-29	1.3	1.3	0
	N_2 (lb/in.)	27.8	59.1	61.2	62.5
	w (in.)	-.0013	-.00131	.000212	.000216

Table 5.2 Maximum Membrane Stress Resultants and Radial Displacement for the Three Point Load Problem (Fig. 5.4)

Meridional Location, θ (degrees)	Stress Resultants	LOADS Program			INDIC=3 Analysis
		m = 5	m = 10	m = 20	
0.0	N_1 (lb/in.)	-3.40	1.53	-.80	-.98
	N_2 (lb/in.)	26.3	31.07	28.9	26.5
	w (in.)	.53	.53	.53	.53
5.0	N_1 (lb/in.)	-1.23	.30	-.39	-.43
	N_2 (lb/in.)	30.0	32.9	32.2	30.6
	w (in.)	.51	.51	.51	.51
60.0	N_1 (lb/in.)	.59	0	-.22	-.02
	N_2 (lb/in.)	49.6	50.5	51.5	50.7
	w (in.)	-.475	-.475	-.475	-.475

Table A.1 Commands Required in Login Command File

```
$ B4AMES   :==SET DEF DRC0:[USER1.B4AMES]
$ B5AMES   :==SET DEF DRC0:[USER1.B5AMES]
$ LOADS    :==SET DEF DRC0:[USER1.LOADS]
$ EDIT     :==EDIT/SOS
$ ASSIGN TT: FOR005
$ ASSIGN TT: FOR006
$ LOGIN    :==@DRC0:[USER1]LOGIN
$ @DRC0:[USER1.B4AMES]B4AMES
$ @DRC0:[USER1.B5AMES]B5AMES
$ @DRC0:[USER1.LOADS]LOADS
```

Table A.2 VAX Command Required to Start B4AMES Program

```
$ SET NOVERIFY
$! Procedure for assigning the PROMPT files prior to interactive
$! runs.
$!
$ LOGIN
$ ASSIGN B4AMES:PROMPT.DAT FOR002
$ ASSIGN B4AMES:PROMPT2.DAT FOR004
$ ASSIGN B4AMES:PROMPT3.DAT FOR007
```

Table A.3 VAX Command Used to Create B4AMES Program Data File

```
$! PURPOSE OF THIS PROCEDURE IS TO ASSEMBLE THE FOR003.DAT FILES
$! FOR A B4AMES BATCH RUN.
$!
$TYPE SYS$INPUT
ALL EXISTING BOSOR.DAT FILES WILL BE DELETED.  SHOULD I PROCEED?
$INQUIRE ANS "Y OR N"
$IF (ANS.EQS."Y") THEN GOTO 5
$TYPE SYS$INPUT
BOSOR FILE ASSEMBLY HALTED
$EXIT
$5:
$DELETE BOSOR.DAT;*
$INQUIRE NSEG "ENTER TOTAL NUMBER OF SEGMENTS IN THE STRUCTURE"
$!
$COPY FOR003.DAT;1 BOSOR.DAT
$ IFILE = 1
$10:
$ IFILE = IFILE + 1
$ IF (IFIL.eGT.NSEG) THEN GOTO 20
$COPY BOSOR.DAT,FOR003.DAT;'IFILE' BOSOR.DAT
$PURGE BOSOR.DAT
$ IF (IFIL.eLT.NSEG) THEN GOTO 10
$20:
$ ILAST = NSEG + 1
$COPY BOSOR.DAT,FOR003.DAT;'ILAST' BOSOR.DAT
$COPY BOSOR.DAT PREP.DAT
$PURGE BOSOR.DAT
$DELETE FOR000.*;*
$DELETE FOR003.DAT;*
$EXIT
```

Table A.4 VAX Command Required to Execute B4AMES Program

```
$ ASSIGN JUNK.DAT SYS$PRINT
$ ASSIGN 'P1'FOR000.DAT FOR000
$ ASSIGN B4AMES:PROMPT.DAT FOR002
$ ASSIGN 'P1'FOR003.DAT FOR003
$ ASSIGN B4AMES:PROMPT2.DAT FOR004
$ ASSIGN 'P1'BOSOR.DAT FOR005
$ ASSIGN 'P1'BOSORLOG.DAT FOR006
$ ASSIGN B4AMES:FOR014.DAT FOR014
$ ASSIGN B4AMES:PROMPT3.DAT FOR007
$ ASSIGN 'P1'BOSORERR.DAT FOR008
$ ASSIGN 'P1'FOR009.DAT FOR009
$ ASSIGN 'P1'For011.DAT FOR011
$ ASSIGN 'P1'FOR040.DAT FOR040
$ ASSIGN 'P1'FOR053.DAT FOR053
$ RUN B4AMES:BOSORREAD
$ RENAME 'P1'BOSORERR.DAT 'P1'BOSOR4OUT.DAT
$ DELETE 'P1'BOSORLOG.DAT;*
$ RUN B4AMES:BOSORMAIN
$ COPY 'P1'BOSOR4OUT.DAT,'P1'BOSORLOG.DAT 'P1'BOSOR4OUT.DAT
$ PURGE 'P1'BOSOR4OUT.DAT
$ DELETE 'P1'BOSORLOG.DAT;*
$ RUN B4AMES:BOSORPOST
$ COPY 'P1'BOSOR4OUT.DAT,'P1'BOSORLOG.DAT 'P1'BOSOR.TEM
$ EDIT 'P1'BOSOR.DAT
EBT
$ COPY 'P1'BOSOR.DAT,'P1'BOSOR.TEM 'P1'B4AMES.OUT
$ DELETE 'P1'BOSOR.TEM;*
$ DELETE 'P1'BOSOR4OUT.DAT;*
$ DELETE 'P1'BOSORLOG.DAT;*
$ DELETE 'P1'BOSORERR.DAT;*
$ DELETE 'P1'FOR000.DAT;*
$ DELETE 'P1'FOR008.DAT;*
$ DELETE 'P1'FOR009.DAT;*
$ DELETE 'P1'FOR011.DAT;*
$ DEASSIGN FOR008
```

Table A.5 VAX Command Required to Execute B4AMES Plotting Routine

```
$ ASSIGN TT: FOR005
$ ASSIGN TT: FOR006
$ RUN B4AMES:BOSORPLOT
$ EXIT
```

Table A.6 VAX Command Required to Execute The LOADS Program

```
$ ASSIGN 'P1'LOADS.DAT FOR005
$ ASSIGN 'P1'LOADS.OUT FOR006
$ ASSIGN 'P1'FOR030.DAT FOR030
$ ASSIGN LOADS:PRESSR.DAT FORPRESSR
$ RUN LOADS:LOADS
```

Table A.7 VAX Command Used to Create B5AMES Main Processor

```
$ LOGIN
$ B5CON
$ RUN B5AMES:MAINSETUP
$ COPY FOR010.DAT MAIN.DAT
```

Table A.8 VAX Command Required to Execute B5AMES Main Processor

```
$ ASSIGN JUNK.DAT SYS$PRINT
$ ASSIGN B5AMES:PROMPT.DAT FOR002
$ ASSIGN 'P1'FOR003.DAT FOR003
$ COPY 'P1'FOR010.DAT 'P1'BOSORMAIN.TEM
$ ASSIGN 'P1'FOR010.DAT FOR005
$ ASSIGN 'P1'BOSORLOG.DAT FOR006
$ ASSIGN 'P1'FOR011.DAT FOR011
$ ASSIGN 'P1'FOR012.DAT FOR012
$ ASSIGN LOADS:PRESSR.DAT;* FOR015
$ PURGE 'P1'FOR011.DAT
$ COPY 'P1'FOR011.DAT FOR011.DAT
$ RENAME 'P1'FOR011.DAT 'P1'LAST11.DAT
$ PURGE FOR011.DAT
$ RUN B5AMES:BOSORMAIN
$ COPY FOR011.DAT 'P1'FOR011.DAT
$ PURGE FOR011.DAT
$ COPY 'P1'BOSORMAIN.TEM,'P1'BOSORLOG.DAT 'P1'BOSORMAIN.OUT
$ DELETE 'P1'BOSORMAIN.TEM;*
$ DELETE 'P1'BOSORLOG.DAT;*
$ DELETE FOR000.DAT;*
$ DELETE 'P1'FOR010.DAT;*
$ EXIT
```

NOTE:

PRESSR.DAT;* IS THE FILE THAT CONTAINS THE PRESSURES CALCULATED FROM THE LOADS PROGRAM. THESE ARE THE TANGENTIAL AND NORMAL PRESSURES AT THE MERIDIAN UNDER CONSIDERATION. THE USER SHOULD SPECIFY THE VERSION NUMBER (THE "*" IN "PRESSR.DAT.*") WHICH INDECATES THE MERDIONAL LOCATION TO BE USED IN THE B5AMES ANALYSIS.

Table A.9 VAX Command Required to Create B5AMES Post Processor

```
$ LOGIN
$ B5CON
$ RUN B5AMES:POSTSETUP
$ COPY FOR010.DAT POST.DAT
```

Table A.10 VAX Command Used to Execute B5AMES Post Processor Interactively

```
$ P1:='F$DIRECTORY()'
$ LOGIN
$ @B5AMES:BOSORPOST.BAT 'P1'
$ ASSIGN TT: FOR005
$ ASSIGN TT: FOR006
$ DEASSIGN SYS$PRINT
```

Table A.11 VAX Command Required to Batch B5AMES Post Processor

```
$ ASSIGN JUNK.DAT SYS$PRINT
$ @ B5AMES:BOSORPOST.BAT 'P1'
```

Table A.12 Input Data for Interactive VAX Version of B4AMES for Modal Vibration Analysis of the Containment in Figure 5.6

```
         2 $ INDIC = analysis type indicator
         2 $ NPRT = output options (1=minimum, 2=medium, 3=maximum)
         0 $ ISTRES= output control (0=resultants, 1=sigma, 2=epsilon)
         3 $ NSEG  = number of shell segments (less than 25)
        33 $ NMESH = number of node points (5 = min.; 98 = max.)( 1)
         3 $ NTYPEH= control integer (1 or 2 or 3) for nodal point spacing
         1 $ NSHAPE= indicator (1,2 or 4) for geometry of meridian
  600.7500$ R1     = radius at beginning of segment (see p. 66)
         0 $ Z1     = axial coordinate at beginning of segment
  600.7500$ R2     = radius at end of segment
       480 $ Z2     = axial coordinate at end of segment
         0 $ IMP   = indicator for imperfection (0=none, 1=some)
         3 $ NTYPEZ= control (1 or 3) for reference surface location
 0.7500000$ ZVAL  = distance from leftmost surf. to reference surf.
     N     $ Do you want to print out r(s), r'(s), etc. for this segment?
         3 $ NRINGS= number (max=20) of discrete rings in this segment
         2 $ NTYPE = control for identification of ring location (2=z, 3=r)
       120 $ Z(I)  = axial coordinate of Ith ring, z( 1)
       240 $ Z(I)  = axial coordinate of Ith ring, z( 2)
       360 $ Z(I)  = axial coordinate of Ith ring, z( 3)
         1 $ NTYPER= type (0 or 1 or 2 or 4 or 5) of discrete ring no.( 1)
         1 $ NTYPER= type (0 or 1 or 2 or 4 or 5) of discrete ring no.( 2)
         1 $ NTYPER= type (0 or 1 or 2 or 4 or 5) of discrete ring no.( 3)
  29000000 $ E      = Young's modulus of ring( 1)
        36 $ A      = cross section area of ring( 1)
      3888 $ Iy     = moment of inertia about y-axis (see fig. on p.70)( 1)
         3 $ Ix     = moment of inertia about x-axis( 1)
         0 $ Ixy    = product of inertia( 1)
  18.75000$ E1     = radial component of ring eccentricity (see p. 70)( 1)
         0 $ E2     = axial  component of ring eccentricity( 1)
 133846152 $ GJ     = torsional rigidity( 1)
 0.7350000E-03 $ RM    = ring material density (e.g. aluminum=.0002535)( 1)
  29000000 $ E      = Young's modulus of ring( 2)
        36 $ A      = cross section area of ring( 2)
      3888 $ Iy     = moment of inertia about y-axis (see fig. on p.70)( 2)
         3 $ Ix     = moment of inertia about x-axis( 2)
         0 $ Ixy    = product of inertia( 2)
  18.75000$ E1     = radial component of ring eccentricity (see p. 70)( 2)
         0 $ E2     = axial  component of ring eccentricity( 2)
 133846152 $ GJ     = torsional rigidity( 2)
 0.7350000E-03 $ RM    = ring material density (e.g. aluminum=.0002535)( 2)
  29000000 $ E      = Young's modulus of ring( 3)
        36 $ A      = cross section area of ring( 3)
      3888 $ Iy     = moment of inertia about y-axis (see fig. on p.70)( 3)
         3 $ Ix     = moment of inertia about x-axis( 3)
         0 $ Ixy    = product of inertia( 3)
  18.75000$ E1     = radial component of ring eccentricity (see p. 70)( 3)
         0 $ E2     = axial  component of ring eccentricity( 3)
 133846152 $ GJ     = torsional rigidity( 3)
 0.7350000E-03 $ RM    = ring material density (e.g. aluminum=.0002535)( 3)
```

```
        0 $ LINTYP= indicator (0, 1, 2 or 3) for type of line loads
        0 $ NLTYPE=control (0,1,2,3) for type of surface loading
        2 $ NWALL = index (1, 2, 4, 5, or 8) for wall construction
 29000000 $ E     = Young's modulus for skin
 0.3000000$ U     = Poisson's ratio for skin
 0.7350000E-03 $ SM =mass density of skin (e.g. alum.=.00025 lb-sec**2/in**4)
        0 $ ALPHA = coefficient of thermal expansion
        0 $ ANRS  = control (0 or 1) for addition of smeared stiffeners
       -1 $ SUR   = control for thickness input (0 or 1 or -1)
        1 $ NTYPET= index (1 or 3) for type of input for thickness
        3 $ NTVALU= number of callouts along segment for thickness
        2 $ NTYPE = control for meaning of thickness callout (2=z, 3=r)
        0 $ Z(I)  = axial coordinate of Ith thickness callout, z( 1)
      465 $ Z(I)  = axial coordinate of Ith thickness callout, z( 2)
      480 $ Z(I)  = axial coordinate of Ith thickness callout, z( 3)
  1.500000$ TVAL(i) = thickness at Ith callout, TVAL( 1)
  1.500000$ TVAL(i) = thickness at Ith callout, TVAL( 2)
        1 $ TVAL(i) = thickness at Ith callout, TVAL( 3)
     Y    $ Do you want to print out ref. surf. location and thickness?
     N    $ Do you want to print out the C(i,j) at meridional stations?
     N    $ Do you want to print out distributed loads along meridian?
       61 $ NMESH = number of node points (5 = min.; 98 = max.)( 2)
        3 $ NTYPEH= control integer (1 or 2 or 3) for nodal point spacing
        1 $ NSHAPE= indicator (1,2 or 4) for geometry of meridian
  600.7500$ R1      = radius at beginning of segment (see p. 66)
      480 $ Z1      = axial coordinate at beginning of segment
  600.7500$ R2      = radius at end of segment
     1200 $ Z2      = axial coordinate at end of segment
        0 $ IMP   = indicator for imperfection (0=none, 1=some)
        3 $ NTYPEZ= control (1 or 3) for reference surface location
 0.7500000$ ZVAL  = distance from leftmost surf. to reference surf.
     N    $ Do you want to print out r(s), r'(s), etc. for this segment?
        3 $ NRINGS= number (max=20) of discrete rings in this segment
        2 $ NTYPE = control for identification of ring location (2=z, 3=r)
      480 $ Z(I)  = axial coordinate of Ith ring, z( 1)
      720 $ Z(I)  = axial coordinate of Ith ring, z( 2)
      960 $ Z(I)  = axial coordinate of Ith ring, z( 3)
        1 $ NTYPER= type (0 or 1 or 2 or 4 or 5) of discrete ring no.( 1)
        1 $ NTYPER= type (0 or 1 or 2 or 4 or 5) of discrete ring no.( 2)
        1 $ NTYPER= type (0 or 1 or 2 or 4 or 5) of discrete ring no.( 3)
 29000000 $ E     = Young's modulus of ring( 1)
       36 $ A     = cross section area of ring( 1)
     3888 $ Iy    = moment of inertia about y-axis (see fig. on p.70)( 1)
        3 $ Ix    = moment of inertia about x-axis( 1)
        0 $ Ixy   = product of inertia( 1)
  18.25000$ E1    = radial component of ring eccentricity (see p. 70)( 1)
        0 $ E2    = axial  component of ring eccentricity( 1)
133846152 $ GJ    = torsional rigidity( 1)
 0.7350000E-03 $ RM    = ring material density (e.g. aluminum=.0002535)( 1)
 29000000 $ E     = Young's modulus of ring( 2)
       36 $ A     = cross section area of ring( 2)
     3888 $ Iy    = moment of inertia about y-axis (see fig. on p.70)( 2)
        3 $ Ix    = moment of inertia about x-axis( 2)
        0 $ Ixy   = product of inertia( 2)
 18.25000$ E1    = radial component of ring eccentricity (see p. 70)( 2)
        0 $ E2    = axial  component of ring eccentricity( 2)
133846152 $ GJ    = torsional rigidity( 2)
 0.7350000E-03 $ RM    = ring material density (e.g. aluminum=.0002535)( 2)
 29000000 $ E     = Young's modulus of ring( 3)
       36 $ A     = cross section area of ring( 3)
     3888 $ Iy    = moment of inertia about y-axis (see fig. on p.70)( 3)
        3 $ Ix    = moment of inertia about x-axis( 3)
        0 $ Ixy   = product of inertia( 3)
 18.25000$ E1    = radial component of ring eccentricity (see p. 70)( 3)
        0 $ E2    = axial  component of ring eccentricity( 3)
133846152 $ GJ    = torsional rigidity( 3)
 0.7350000E-03 $ RM    = ring material density (e.g. aluminum=.0002535)( 3)
        0 $ LINTYP= indicator (0, 1, 2 or 3) for type of line loads
        0 $ NLTYPE=control (0,1,2,3) for type of surface loading
        2 $ NWALL = index (1, 2, 4, 5, or 8) for wall construction
 29000000 $ E     = Young's modulus for skin
 0.3000000$ U     = Poisson's ratio for skin
 0.7350000E-03 $ SM =mass density of skin (e.g. alum.=.00025 lb-sec**2/in**4)
        0 $ ALPHA = coefficient of thermal expansion
        0 $ ANRS  = control (0 or 1) for addition of smeared stiffeners
       -1 $ SUR   = control for thickness input (0 or 1 or -1)
        1 $ NTYPET= index (1 or 3) for type of input for thickness
        2 $ NTVALU= number of callouts along segment for thickness
        2 $ NTYPE = control for meaning of thickness callout (2=z, 3=r)
      480 $ Z(I)  = axial coordinate of Ith thickness callout, z( 1)
     1200 $ Z(I)  = axial coordinate of Ith thickness callout, z( 2)
        1 $ TVAL(i) = thickness at Ith callout, TVAL( 1)
        1 $ TVAL(i) = thickness at Ith callout, TVAL( 2)
 N        $ Do you want to print out ref. surf. location and thickness?
 N        $ Do you want to print out the C(i,j) at meridional stations?
 N        $ Do you want to print out distributed loads along meridian?
       59 $ NMESH = number of node points (5 = min.; 98 = max.)( 3)
        1 $ NTYPEH= control integer (1 or 2 or 3) for nodal point spacing
        2 $ NHVALU= number of callouts for nodal point spacing
        1 $ IHVALU(I)= Ith callout for nodal point spacing, IHVALU( 1)
       58 $ IHVALU(I)= Ith callout for nodal point spacing, IHVALU( 2)
       15 $ HVALU=meridional arc length between nodal points,HVALU( 1)
 10.70000$ HVALU=meridional arc length between nodal points,HVALU( 2)
        4 $ NSHAPE= indicator (1,2 or 4) for geometry of meridian
        4 $ NST   = indicator for geometry: 1=general, 4=ellipsoid
 300.7500$ ZMAX  = length of half of axial axis of ellipsoid (p.66)
 600.7500$ XMAX  = length of half of radial axis of ellipsoid
 300.7500$ ZA    = axial coordinate to beginning of segment (p.66)
 601.4593$ ZB    = axial coordinate to end of segment (p. 66)
        0 $ ALPHAT= radial coordinate from axis of rev. to center.
 N        $ Do you want to print out the (z,r) coordinates (input data)?
 N        $ Do you want to print out debugging quantities for geometry?
        0 $ IMP   = indicator for imperfection (0=none, 1=some)
        3 $ NTYPEZ= control (1 or 3) for reference surface location
 0.7500000$ ZVAL  = distance from leftmost surf. to reference surf.
 N        $ Do you want to print out r(s), r'(s), etc. for this segment?
        0 $ NRINGS= number (max=20) of discrete rings in this segment
        0 $ LINTYP= indicator (0, 1, 2 or 3) for type of line loads
        0 $ NLTYPE=control (0,1,2,3) for type of surface loading
        2 $ NWALL = index (1, 2, 4, 5, or 8) for wall construction
 29000000 $ E     = Young's modulus for skin
 0.3000000$ U     = Poisson's ratio for skin
 0.7350000E-03 $ SM =mass density of skin (e.g. alum.=.00025 lb-sec**2/in**4)
        0 $ ALPHA = coefficient of thermal expansion
        0 $ ANRS  = control (0 or 1) fcr addition of smeared stiffeners
       -1 $ SUR   = control for thickness input (0 or 1 or -1)
        1 $ NTYPET= index (1 or 3) for type of input for thickness
        3 $ NTVALU= number of callouts along segment for thickness
        2 $ NTYPE = control for meaning of thickness callout (2=z, 3=r)
        0 $ Z(I)  = axial coordinate of Ith thickness callout, z( 1)
       15 $ Z(I)  = axial coordinate of Ith thickness callout, z( 2)
  300.7500$ Z(I)  = axial coordinate of Ith thickness callout, z( 3)
        1 $ TVAL(i) = thickness at Ith callout, TVAL( 1)
  1.500000$ TVAL(i) = thickness at Ith callout, TVAL( 2)
  1.500000$ TVAL(i) = thickness at Ith callout, TVAL( 3)
    N     $ Do you want to print out ref. surf. location and thickness?
    N     $ Do you want to print out the C(i,j) at meridional stations?
    N     $ Do you want to print o u t distributed loads along meridian?
        0 $ NLAST = plot options  (-1=none, 0=geometry, 1=u,v,w)
      -10 $ NOB   = starting number of circ. waves (buckling analysis)
      -10 $ NMINB = minimum number of circ. waves  (buckling analysis)
        0 $ NMAXB = maximum number of circ. waves  (buckling analysis)
        1 $ INCRB = increment in number of circ. waves (buckling)
       20 $ NVEC  = number of eigenvalues for each wave number
        0 $ P      = pressure or surface traction multiplier
        0 $ TEMP   = temperature rise multiplier
      0 $ OMEGA  = angular vel. about axis of revolution (rad/sec)
      0 $ Number of poles (places where r=0) in SEGMENT( 1)
      1 $ At how many stations is this segment constrained to ground?
      1 $ INODE = nodal point number of constraint to ground, INODE( 1)
      1 $ IUSTAR= axial displacement (0=free, 1=constrained)
      1 $ IVSTAR= circumferential displacement (0=free, 1=constrained)
      1 $ IWSTAR= radial displacement (0=free, 1=constrained)
      1 $ ICHI  = meridional rotation (0=free, 1=constrained)
      0 $ D1    = radial component of offset of ground support
      0 $ D2    = axial component of offset of ground support
  Y     $ Is this constraint the same for both prebuckling and buckling?
  N     $ Is this segment joined to any lower-numbered segments?
      0 $ Number of poles (places where r=0) in SEGMENT( 2)
      0 $ At how many stations is this segment constrained to ground?
  Y     $ Is this segment joined to any lower-numbered segments?
      1 $ At how may stations is this segment joined to previous segs.?
      1 $ INODE = node in current segment (ISEG) of junction, INODE( 1)
      1 $ JSEG  = segment no. of previous segment involved in junction
     33 $ JNODE = node in previous segmnt (JSEG) of junction
      1 $ IUSTAR= axial displacement (0=not slaved, 1=slaved) .
      1 $ IVSTAR= circumferential displacement (0=not slaved, 1=slaved)
      1 $ IWSTAR= radial displacement (0=not slaved, 1=slaved)
      1 $ ICHI  = meridional rotation (0=not slaved, 1=slaved)
      0 $ D1    = radial component of juncture gap
      0 $ D2    = axial component of juncture gap
  Y     $ Is this constraint the same for both prebuckling and buckling?
```

```
   0 $ Number of poles (places where r=0) in SEGMENT( 3)
   1 $ At how many stations is this segment constrained to ground?
  59 $ INODE = nodal point number of constraint to ground, INODE( 1)
   0 $ IUSTAR= axial displacement (0=free, 1=constrained)
   0 $ IVSTAR= circumferential displacement (0=free, 1=constrained)
   0 $ IWSTAR= radial displacement (0=free, 1=constrained)
   0 $ ICHI  = meridional rotation (0=free, 1=constrained)
   0 $ D1    = radial component of offset of ground support
   0 $ D2    = axial component of offset of ground support
Y    $ Is this constraint the same for both prebuckling and buckling?
Y    $ Is this segment joined to any lower-numbered segments?
   1 $ At how may stations is this segment joined to previous segs.?
   1 $ INODE = node in current segment (ISEG) of junction, INODE( 1)
   2 $ JSEG  = segment no. of previous segment involved in junction
  61 $ JNODE = node in previous segmnt (JSEG) of junction
   1 $ IUSTAR= axial displacement (0=not slaved, 1=slaved)
   1 $ IVSTAR= circumferential displacement (0=not slaved, 1=slaved)
   1 $ IWSTAR= radial displacement (0=not slaved, 1=slaved)
   1 $ ICHI  = meridional rotation (0=not slaved, 1=slaved)
   0 $ D1    = radial component of juncture gap
   0 $ D2    = axial component of juncture gap
   Y $ Is this constraint the same for both prebuckling and buckling?
   Y $ Do you want to list output for segment( 1)
   Y $ Do you want to list output for segment( 2)
   Y $ Do you want to list output for segment( 3)
   N $ Do you want to list prebuckling resultants and ring forces?
   N $ Do you want to list forces in the discrete rings, if any?
```

Table A.13 Listing of the LOADS Program

```
      DIMENSION TITLE(18),ARCL(500)
      COMMON/SEGM/R(500),NMP(25),PN(100),NSEG
      COMMON/FF/DS(500),DH(500),GM(250)
      COMMON/INT/FE(250,100),FR(250),NTT,NMODES,NMPT,T(100),DAMRAT
      COMMON/MEMBRN/N1(500,250),N2(500,250),N12(500,250)
      COMMON/MOMENT/M1(500,250),M2(500,250),M12(500,250)
      COMMON/DISP/U(500,250),V(500,250),W(500,250),THETA(36)
      COMMON/VIBRAT/NVEC(50),NM,NSTART,NFIN,INCR,NMERD
      COMMON/GEOM/SMPA(500),CURV1(500),STA(500)
      REAL N1,N2,N12,M1,M2,M12
      CHARACTER*4 TITLE
      DOUBLE PRECISION DS,DH
C
C  IMPORTANT NOTES:
C =================
C    THE FOLLOWING IS TO ILLUSTRATE TO THE USER HOW THE NODAL POINT
C    SYSTEM IS HANDELED IN BOSOR4/5 AND THIS PROGRAM.
C
C  X-------------X-------------X----------------------------X-----
C  *-+-*----+----*------+------*-------------+-------------*-----
C  #-----#--------#---------------#-------------------#---------
C
C  X    = MESH POINTS PROVIDED BY THE USER.
C  *    = MESH POINTS CREATED INSIDE BOSOR4/5 (SEE PAGE 23 AND 63 IN
C         BOSOR4 MANUAL). THESE ARE CALLED W-POINTS IN THE BOSOR4 MANUAL.
C         THE NORMAL DISPLACEMENTS ARE CALCULATED AT THESE W-POINTS. IN
C         THE BOSOR4/5 PROGRAMS TWO EXTRA MESH POINTS ARE INSERTED AT
C         DISTANCE A=B/20 FROM EACH SEGMENT ENDS,WHERE B IS THE SPACING
C         BETWEEN THE MESH POINTS AT BOTH ENDS.THE PURPOSE OF THESE
C         ADDITIONAL POINTS IS TO REDUCE TRUNCATION ERROR AT BOUNDERIES
C         OF SHELL SEGMENTS AND TO PREVENT SPURIOUS BUCKLING OR
C         VIBRATION MODES(SEE BOSOR4 MANUAL PAGE 63).
C  +    = THESE ARE CALLED U AND V POINTS AND ARE LOCATED MIDWAY
C         BETWEEN THE W-POINTS. THE MERIDIONAL AND CIRCUMFERENTIAL
C         DISPLACEMENTS;RESPECTIVELY; ARE CALCULATED AT THESE U AND V
C         POINTS.
C  #    = WHERE THE SHELL ENERGY DENSITY IS EVALUATED. THESE POINTS
C         ARE LOCATED MIDWAY BETWEEN THE U AND V POINTS. ALL THE RESULTS
C         i.e. DISPLACEMENTS AND STRESS RESULTANTS ARE PRINTED AT THESE
C         ENERGY POINTS. THEY ARE CALLED STATION POINTS IN BOSOR4/5
C         MANUALS.
C
C  READ IN THE FOLLOWING CONTROL INFORMATION:
C ==========================================
C
C          1-NMERD  = NUMBER OF CIRCUMFERENTIAL STATION TO BE
C                     USED FOR CHECKING AND PRINTING OUT THE STRESS
C                     RESULTANT ENVELOPE. NOTE THIS NUMBER HAS TO BE
C                     SPECIFIED PER THE 360 DEGREES CIRCUMFERENCE.
C
C          2-NPRESS = EQUIVALENT PRESSURE CALCULATION FREQUENCY.
```

```
C
C          3-NPSTRE = STRESS RESULTANTS CALCULATION FREQUENCY.
C
C          4-NASME  = STRESS INTENSITY ENVELOPE UPDATE FREQUENCY.
C
C          5-NPRINT = PRINT OUT OPTION ( 0=MINIMUM  1=MAXIMUM).
C
C          6-NSTRT  = RESTART OPTION ( 0=NEW CASE  1=CONTINUE THE
C                     ANALYSIS OF A PREVIOUSLY ANALYZED PROBLEM).
C
C          7-NDTOP  = TIME STEP SIZE AND INTERATION POINTS OPTION.
C                      0= DT IS 0.1* THE SMALLEST NATURAL PERIOD.
C                      1= DT AND THE NUMBER OF THE INTEGRATION
C                         POINTS ARE PROVIDED BY THE USER.
C
C          8-DAMRAT = DAMPING ,FOR EXAMPLE,IF A 5% DAMPING IS USED
C                     THEN DAMRAT IS INPUT AS 0.05 .
C
C          9-ROW    = SHELL SKIN MATERIAL MASS DENSITY.
C
      READ(5,*)NMERD,NPRESS,NPSTRE,NASME,NPRINT,NSTRT,NDTOP,DAMRAT,ROW
      READ(5,*)(THETA(I),I=1,NMERD)
C
C   THE FOLLOWING IS THE RESULTS OF INDIC2 ANALYSIS ON FILE 14.
C
      WRITE(6,*) '*** RECOVER BOSOR4 OUTPUT'
      READ(14) (TITLE(I),I=1,18)
C   NSEG IS NUMBER OF SEGMENTS
      READ(14) NSEG
      IX=0
      DO 130 I=1,NSEG
C   NMP(I) IS NUMBER OF MESHPOINTS FOR SEGMENT I
      READ(14) NMP(I)
      DO 140 J=1,NMP(I)
      IX=IX+1
C
C   STA IS STATION
C   R IS RADIUS
C   SMPA IS THE MASS PER AREA
C
      READ(14) STA(IX),R(IX),CURV1(IX)
 140  CONTINUE
 130  CONTINUE
C
C    READ IN :
C           1-NSTART = STARTING NUMBER OF CIRC. WAVES
C           2-NFIN   = ENDING NUMBER OF CIRC. WAVES
C           3-INCR   = STEP IN NUMBER OF CIRC. WAVES
C
      READ(14)NSTART,NFIN,INCR
      NSTEPA=IABS(INCR)
      NM=(IABS(NFIN-NSTART)/NSTEPA)+1
C   NMPT IS THE TOTAL NUMBER OF MESH IN THE STRUCTURE.THIS INCLUDES
C   THE EXTRA TWO POINTS ADDED INSIDE THE BOSOR4 PRAM (SEE BOSOR4
C   MANUAL PAGE 63)
C
      NMPT=0
      DO 135 I=1,NSEG
  135 NMPT= NMPT+NMP(I)
      IF (NMPT.LE.500) GOTO 137
      WRITE(6,*) 'NUMBER OF MESH POINTS EXCEEDS 500....TOTAL IS ',
     $NMPT,'  INCREASE ARRAY SIZES TO CONTINUE'
      STOP
  137 CONTINUE
C   NVEC(N) IS THE NUMBER OF MERIDIONAL MODES
      NMODES=0
      JK=0
      DO 100 N=1,NM
      READ(14) NVEC(N)
      NMODES=NMODES+NVEC(N)
      DO 160 J=1,NVEC(N)
      JJ=JK+J
      DO 170 K=1,NMPT
C   U IS MERIDIONAL DISPACEMENT
C   V IS CIRCUMFERENTIAL DISPLACEMENT
C   W IS NORMAL DISPLACEMENT
C   N1 IS MERIDIONAL STRESS RESULTANT
C   N2 IS CIRCUMFERENTIAL STRESS RESULTANT
C   N12 IS SHEAR STRESS RESULTANT
C   M1 IS MERIDIONAL MOMENT RESULTANT
C   M2 IS CIRCUMFERENTIAL MOMENT RESULTANT
C   M12 IS TORSIONAL MOMENT RESULTANT
C
C DEFINITION OF DS,DK AND DH
C ==========================
C  *  = W-POINTS
C
C    X-----------------X-----------------------X
C    /------- DK ------/----------- DH ---------/
C             <---------- DS -------->
C
C           DS=(DK+DH)/2.
C
C NOTICE THAT DS IS THE DISTANCE CENTER TO CENTER BETWEEN THE W-POINTS
C
C
      READ(14) U(K,JJ),V(K,JJ),W(K,JJ),N1(K,JJ),N2(K,JJ),
     1N12(K,JJ),M1(K,JJ),M2(K,JJ),M12(K,JJ),SMPA(K),DS(K),DH(K)
  170 CONTINUE
C   FR(JJ) IS FREQUENCY (HZ) FOR MODE
C   GM(JJ) IS GENERALIZED MASS FOR MODE
      READ(14) FR(JJ),GM(JJ)
  160 CONTINUE
      JK=NMODES
  100 CONTINUE
C
C   ARCL IS THE ARC LENGTH MEASURED FROM THE FIRST NODAL POINT
C
```

```
      ARCL(1)=0
      IX=0
      DO 141 I=1,NSEG
      DO 141 J=1,NMP(I)
      IX=IX+1
      IF (I.GT.1.AND.J.EQ.1) ARCL(IX)=ARCL(IX-1)
      IJ=IX+1
 141  ARCL(IJ)=ARCL(IX)+2*DS(IX)-DH(IX)
C
      IF(NMODES.LE.250) GO TO 139
C
      WRITE(6,*) 'NUMBER OF MODES  EXCEEDS 250....TOTAL IS ',
     1 NMODES,'  INCREASE ARRAY SIZES TO CONTINUE'
      STOP
C
C
C  SEARCH FOR THE MAXIMUM NATURAL FREQUENCY,i.e,THE SMALLEST NATURAL
C  PERIOD WHICH WILL BE USED TO DETERMINE THE TIME INTEGRATION STEP
C  SIZE.
C
139   FMI  =0
      FMAX =1.E+20
      PI=3.141592654
      DO 51  I=1,NMODES
      OMG =2.*PI*FR(I)
      IF (OMG.LE.FMAX)  FMAX=OMG
 51   IF (OMG.GE.FMI)   FMI=OMG
      IF(NDTOP.EQ.1) GO TO 149
C
C  STEP SIZE"DT" OF 0.1* THE SMALLEST NATURAL PERIOD WILL BE USED
C  NT1  = TOTAL NUMBER OF TIME STEPS TO BE USED IN THE NUMERICAL
C  INTEGRATION .
C
      DT=(2.*PI/FMI)/10.
      NT1=INT((((T(NTT)-TIMSTR)+2.*PI/FMAX)/DT)+1)
C
      GO TO 159
 149   CONTINUE
C
C TIME STEP SIZE AND THE INTEGRATION POINTS NUMBER ARE TO BE PROVIDED BY
C THE USER SINCE THE NDTOP IS SPECIFIED ONE ON THE CONTROL INFORMATION
C LINE.
C
C READ IN:
C             1- DT  = TIME STEP SIZE.
C             2- NT1 = NUMEBR OF INTEGRATION OINTS.
C
C
      READ(5,*)DT,NT1
C
C   WRITE ALL DATA IN FORMATTED FORM TO FILE FOR006.DAT
C   ****************************************************
C
159   WRITE(6,1000) (TITLE(I),I=1,18)
```

```
      WRITE(6,2000)NSEG,NMPT,NSTART,NFIN,INCR,NMERD,DAMRAT,NPRESS,
     $NPSTRE,NASME,NPRINT,NSTRT,NDTOP,DT,NT1
C
C  IF THE PRINT OUT OPTION IS SPECFIED ON THE CONTROL LINE TO BE MAXIMUM
C  PRINT OUT THE RESULTS OF BOSOR4 INDIC=2 ANALYSIS.
C
      IF(NPRINT.EQ.0) GO TO 230
      IX=0
      WRITE(6,11)FMI,FMAX
      WRITE(6,2005)
      DO 210 I=1,NSEG
      WRITE(6,2010)NMP(I),I
      WRITE(6,2020)
      DO 220 J=1,NMP(I)
      IX=IX+1
      WRITE(6,2030)J,STA(IX),ARCL(IX),R(IX),SMPA(IX),CURV1(IX)
  220 CONTINUE
      WRITE(6,2040)
  210 CONTINUE
      WRITE(6,2050)
      LL=0
      DO 230 N=1,NM
      NN=NSTART+(N-1)*INCR
      WRITE(6,2060)NN,NVEC(N)
      DO 240 I=1,NVEC(N)
      LL=LL+1
      IX=0
      WRITE(6,2070)I,FR(LL),GM(LL)
      IX=0
      DO 250 J=1,NSEG
      WRITE(6,2080)J
      DO 260 K=1,NMP(J)
      IX=IX+1
      WRITE(6,2090)K,U(IX,LL),V(IX,LL),W(IX,LL),N1(IX,LL),N2(IX,LL),
     $N12(IX,LL),M1(IX,LL),M2(IX,LL),M12(IX,LL)
  260 CONTINUE
  250 CONTINUE
      WRITE(6,3000)
  240 CONTINUE
  230 CONTINUE
      WRITE(6,3050)
C
C READ IN THE TIME , CIRCUMFERENTIAL VARIATION AND MERDIONAL VARIATION
C OF THE APPLIED PRESSURE. CALCULATE THE FOURIER AMPLITUDES AND THE
C EQUIVALENT MODAL FORCE VECTOR.
C
      CALL  READIT(NPRINT)
C
C
C PERFORM THE FOLLOWING STEPS:
C       1- SOLVE THE UNCOUPLED DIFFERENTIAL EQUATION OF MODAL COORDINATE EQUATION.
C       2- CALCULATE THE STRESS RESULTANTS  A THE DIFFERENT MERDIONAL
C          LOCATION (NMERD) EVERY NPSTRE . THIS IS PERFORMED IN SUB-
C          ROUTINE STRESS.
```

```
C      3- FIND AND UPDATE THE STRESS INTENSITES ENVELOPS AT ALL NMERD.
C         THIS IS ACCOMPLISHE IN SUBRUTINE ASME EVERY NASME.
C
C      4- CLACULATE THE EQIVALENT AXISYMMETRIC PRESSURES EVERY NPRESS
C         FOR ALL NMERD. THIS ARE TO BE USED IN BOSOR5 ANALYSIS. SUB-
C         ROUTINE PRECAL PERFORMES THIS STEP.
C
       CALL SOLVE (NPRESS,NPSTRE,NSTRT,NASME,NPRINT,NT1,DT,ROW)
C
C  FORMAT STATMENTS USED IN INPUT THE DATA OR PRINT OUT THE RESULTS
C
 11    FORMAT(/,5X,' LARGEST MODAL FREQUENCY',19X,':',F10.3,'  CPS',
      $/,5X,' SMALLEST MODAL FREQUENCY',18X,':',F10.3,'  CPS',/)
45     FORMAT(15X,F8.3,13X,F6.3,15X,F8.3,13X,F6.3)
112    FORMAT(3E20.10)
304    FORMAT(//,5X,'EQUIVALENT MODAL FORCE VECTOR AT TIME =',1PE12.5,
      $//,10X,'CIRCUMFERENTIAL WAVE',10X,'MODE NUMBER',
      $10X,'EQUIVALENT FORCE',//)
1000   FORMAT(////,' PROBLEM TITLE:  ',18A4/17X,18A4//)
 306   FORMAT(16X,I5,22X,I3,16X,1PE12.5)
2000   FORMAT(5X,' TOTAL NUMBER OF SEGMENTS                     : ',I6,/,5X,
      1' TOTAL NUMBER OF MESH POINTS IN MODEL       : ',I6,/,5X,
      2' STARTING NUMBER OF CIRCUMFERENTIAL WAVES  : ',I6,/,5X,
      3' ENDING NUMBER OF CIRCUMFERENTIAL WAVES    : ',I6,/,5X,
      4' STEP IN NUMBER OF CIRCUMFERENTIAL WAVES   : ',I6,/,5X,
      5' NUMBER OF CIRCUMFERENTIAL STATIONS      ',/,5X,
      6' SPECIFIED FOR PRINT OUTPUT RESULTS        : ',I6,/,5X,
      7' DAMPING RATIO USED IS                     : ',F6.3,/,5X,
      8' EQUIVALENT PRESSURES ARE CALCULATED EVERY : ',I6,' TIME STEPS',
      8/,5X,' STRESS RESULTANTS ARE CALCULATED EVERY    : ',I6,
      8' TIME STEPS',/,5X,' STRESS INTENSITY ENVELOPE IS UPDATED EVERY'
      8,':',I7,' TIME STEPS',/,5X,' PRINT OUT  OPTION (0=MINIMUM ',
      8'  1=MAXIMUM ): ',I6,/,5X,
      8' RESTART OPTION (1=RESTART 0=NEW CASE)     : ',I6,/,
      85X,' INTEGRATION POINTS AND STEP SIZE OPTION   : ',I6,/,5X,
      8' TIME STEP SIZE                            : ',1PE12.5,/,
      85X,' NUMBER OF INTEGRATION POINTS                : ',I6,///)
2005   FORMAT(5X,14H THE FOLLOWING,
      152H MESH POINT NUMBERING INCLUDES THE EXTRA MESH POINTS,/,5X,
      158H**********************************************************,
      $8H********,//)
2010   FORMAT(' THERE ARE ',I3,' MESH POINTS IN SEGMENT ',I2,/)
2020   FORMAT(' MESH POINT    STATION         ARC LENGTH ',
      $'       RADIUS         SMPA         CURVARTURE')
2030   FORMAT(4X,0PI3,4X,1P5E15.7)
2040   FORMAT(///)
2050   FORMAT(////,'**** RESULTS FOR EACH MODE *****',///)
2060   FORMAT(' ------------------------',/,1X,' CIRCUMFERENTIAL WAVE :',
      $I2,/,1X,I2,'EIGENVECTORS FOUND  :',/,'------------------------',/)
2070   FORMAT(' EIGENVECTOR (MODE SHAPE)   :' 0PI15,/,' EIGENVALUE ( FREQ',
      $'UNCY (HZ):',1PE15.7,/,' EQUIVALENT MASS              :',1PE15.7,//)
2080   FORMAT(' SEGMENT: ', I2,/,' MESH',/,' POINT',4X,'U',14X,'V',
      $12X,'W',12X,'N1',12X,'N2',12X,'N12',11X,'M1',11X,'M2',13X,'M12',/)
2090   FORMAT(0PI3,9(2X,1PE12.5))
```

```
3000  FORMAT(////)
3050  FORMAT('1')
       STOP
       END
C
C*********************
C* DECK= READIT      *
C*********************
C
C
       SUBROUTINE READIT (NPRINT)
C
C THIS SUBROUTINE READS IN THE INFORMATION NEEDED TO DEFINE THE TIME,
C MERIDIONAL AND CIRCUMFERENTIAL VARIATION OF THE APPLIED PRESSURE.
C SEVERAL SUBROUTINE ARE CALLED WITHIN THIS ROUTINE. FOR MORE
C INFORMATION ABOUT EACH OF THESE SUBROUTINE PLEASE SEE EACH OF THESE
C ROUTINE.
C
       DIMENSION ISEG(25),NX(25)
       COMMON XPLUS(100),XMINUS(100),YPLUS(100),YMINUS(100)
       COMMON/SEGM/R(500),NMP(25),PN(100),NSEG
       COMMON/FC/A(25,50),ISEGE,NZ,FN(500,50)
       COMMON/FF/DS(500),DH(500),GM(250)
       COMMON/INT/FE(250,100),FR(250),NTT,NMODES,NMPT,T(100),DAMRAT
       COMMON/MEMBRN/N1(500,250),N2(500,250),N12(500,250)
       COMMON/MOMENT/M1(500,250),M2(500,250),M12(500,250)
       COMMON/DISP/U(500,250),V(500,250),W(500,250),THETA(36)
       COMMON/VIBRAT/NVEC(50),NM,NSTART,NFIN,INCR,NMERD
       COMMON/GEOM/SMPA(500),CURV1(500),STA(500)
       REAL N1,N2,N12,M1,M2,M12
       CHARACTER*4 TITLE
       DOUBLE PRECISION DS,DH
C
C    READ IN :
C          1- NSEGL = NUMBER OF LOADED SEGMENTS
C          2- NTT   = NUMBER OF POINTS USED TO DEFINE PRESSURE-TIME
C                     RELATIONSHIP
C
       READ (5,*)NSEGTT
C  READ IN PRESSURE FOR LOADED SEGMENTS ONLY
C    T(J) = TIME T SUB.J
C
C
       DO 5 JT=1,NTT
       READ (5,*)T(JT)
       T1=T(JT)
C
C   READ IN FOR ALL THE LOADED SEGMENTS:
C
C          1- ISEG(I) = LOADED SEGMENT NUMBER
C          2- NX(I)   = NUMBER OF CICUMFERENTIAL POINTS FOR SPECIFICA-
C                       TION OF THE PRESSURE VARIATION.
C
       DO 10 I=1,NSEGL
```

```
      READ(5,*)ISEG(I),NX(I)
      NZ=NX(I)
      ISEGE=ISEG(I)
C
C  READ AND PRINT OUT IN THE CIRCUMFERENTAIL VARIATION OF THE APPLIED
C PRESSURE.
      CALL CIRCUM (T1,XX,NPRINT)
C
C
C READ IN THE MERDIONAL VARIATION OF THE APPLIED PRESSURE.
C
      CALL PRESSURE (T1,ISEGE,NPRINT)
C
C
C  CALCULATE FOURIER AMPLITUDES AND THE NODAL FORCE VECTORS
C  FOR N-FOURIER TERMS (N RANGE FROM NSTART TO NFIN IN INCR)
C
      N=NSTART
      JK=0
      DO 150 IS=1,NM
      JK=JK+1
C  CALCULAT FOURIER AMPLITUDES IN SUBROUTINE FCOEF.
C
      CALL AMPLIT (JK,N,XX)
C
C  CALCULATE MESH POINTS FORCE VECTOR.
C
      CALL FORCE (N,T1,JK,NPRINT)
      N = N+INCR
150   CONTINUE
10    CONTINUE
C
C
C  CALCULATE THE EQUIVALENT MODAL FORCE VECTOR  FE(J,JT);WHERE
C        I = MODE NUMBER
C       JT = AT TIME POINT NUMBER
C
C
C   INITIALIZIATION OF THE EQUIVELANT MODAL FORCE VECTOR FE.
C
      DO 21 I=1,NMODES
 21   FE(I,JT)=0.0
C
     PI=3.141592654
     NN=NSTART
     I =0
     DO 301 N=1,NM
     DO 302 II=1,NVEC(N)
     I=I+1
     DO 303 K =1,NMPT
303  FE(I,JT)=FE(I,JT)+PI*W(K,I)*FN(K,N)/GM(I)
302  CONTINUE
     NN =NN+INCR
301  CONTINUE
```

```
 5    CONTINUE
      RETURN
      END
C
C*****************
C* DECK= CIRCUM  *
******************
C
C
          SUBROUTINE CIRCUM (T1,XX,NPRINT)
C
C
       COMMON XPLUS(100),XMINUS(100),YPLUS(100),YMINUS(100)
       COMMON/FC/A(25,50),ISEGE,NZ,FN(500,50)
C
C
C  READ IN THE VARIATION OF THE APPLIED PRESSURE IN THE CIRCUMFERENTIAL
C  DIRECTION.THE FOLLOWING OPTION IS THAT USED IN BOSOR4 WITH NODD=3,
C  (SEE BOSOR4 MANUAL PAGE 77).THE USER MUST PROVIDE THE PRESSURE
C  AMPLITUDE YPLUS(THETA) & YMINUS(THETA) AT ANGLES EQUAL PLUS THETA
C  AND MINUS THETA. XPLUS(I1)=THETA + ;XMINUS(I1)=THETA - ;J=1,NX(I)
       DO 26 I1=1,NZ
26     READ(5,*)XPLUS(I1),YPLUS(I1),XMINUS(I1),YMINUS(I1)
C PRINT OUT THE PROVIDED DATA IF NPRINT=1
C
      IF (NPRINT.EQ.0) GO TO 102
      WRITE(6,100) ISEGE,T1
      DO 210 I=1,NZ
210   WRITE (6,200)I,XPLUS(I),YPLUS(I),XMINUS(I),YMINUS(I)
102    DO 55 I1=1,NZ
C
C  CHANGE THE PROVIDED ANGELS TO RADIAL COORDINATES
C
       XPLUS(I1)=XPLUS(I1)*3.1415927/180
 55    XMINUS(I1)=XMINUS(I1)*3.1415927/180.
       XX=XPLUS(NZ)
C
C FORMAT USED
C
100   FORMAT(/,' CIRCUMFERENTIAL DISTRIBUTION OF NORMAL PRESSURE'
     1' APPLIED ON  SEGMENT :',I5,5X,' AT TIME :',F10.5,/)
200   FORMAT(10H CIRC. STA    I3,24H CIRC. COORD.(DEGREES) =  E10.3,
     118H INPUT LOAD VALUE=   E10.3, 14H CIRC. COORD.=    E10.3,
     218H INPUT LOAD VALUE=    E10.3)
      RETURN
      END
C
C******************
C* DECK= PRESSURE *
C******************
C
C
       SUBROUTINE PRESSURE (T1,ISEGE,NPRINT)
C
```

```
C THIS SUBROUTINE READS IN THE MERIDIONAL VARIATION OF THE APPLIED
C PRESSURE.
C
       COMMON/SEGM/R(500),NMP(25),PN(100),NSEG
       COMMON/GEOM/SMPA(500),CURV1(500),STA(500)
C
C  INITIALIZIATION OF THE MESH POINTS PRESSURE VECTOR PN(I);I
C  I=1,NMP(ISEGE).
C
      IX=0
      DO 24 L=1,NMP(ISEGE)
24    PN(L)=0
C
C  READ IN PRESSURE. THE PROGRAM WILL GENERATE VALUES OF PRESSUE FOR
C  THE MISSING MESH POINTS. THE USER HAS TO SPECIFY PRESSURE VALUE
C  AT THE FIRST AND LAST NODES. AN ERROR MESSAGE WILL BE PRINTED
C  AND THIS WILL STOP THE SOLUTION.
C
C  PJ1   = APPLIED PRESSURE AT W-POINT J1.THIS IS PROVIDED BY THE USER.
C  PN(IY)= GENERATED PRESSURE AT STATION POINT IY;WHERE,IY IS A COUNTER
C          TAKES INTO ACCOUNT THE ADDITIONAL TWO POINTS INSERTED FOR
C          EACH SEGMENT BY THE BOSOR4 PROGRAM. IY=1,TOTAL POINTS IN THE
C          MODEL,i.e,IY=1,NMPT.
C  KPJ   = IS THE NUMBER OF THE W POINTS IN THE SEGMENT.
C  IX    = IS THE TOTAL NUMBER OF MESH POINTS INCLUDING THE EXTRA TWO
C          POINTS) STARTING FORM (SEGMENT #1)TO THE (NUMBER OF THE
C          CURRENT SEGMENT - 1). IX = 0 IF THE CURRENT SEGMENT IS #1.
C  L     = IS A COUNTER TAKES CARE OF THE PROVIDED PRESSURE.THIS IS
C          USED TO CHECK IF A PRESSURE IS SPECIFIED AT THE FIRST AND
C          LAST W-POINT.
C  DB    = CALCULATED SLOPE OF THE APPLIED PRESSURE BETWEEN THE CURRENT
C          POINT AND THE PREVIOUSLY SPECIFIED POINT.
      IF (ISEGE.EQ.1) GO TO 53
      IZZ=ISEGE-1
      DO 51 I1=1,IZZ
   51 IX=IX+NMP(I1)
   53  KPJ=NMP(ISEGE)-2
      WRITE(6,30)ISEGE,T1
      LL=IX
      L=0
      IY=1
54    READ(5,*)J1,PJ1
      WRITE(6,35)J1,PJ1
      IF(J1.GT.1.AND.J1.LT.KPJ) IY=J1+1
      IF(J1.EQ.KPJ) IY=J1+2
      IS=IX+IY
      PN(IY)=PJ1
      IF(L.EQ.0) GO TO 85
      DP=(PN(IY)-PN(L))/(STA(IS)-STA(LL))
 85   L=L+1
      LL=LL+1
      IF(IY-L)103,86,83
 83   PN(L)=PN(L-1)+DP*(STA(LL)-STA(LL-1))
      GO TO 85
 86     IF(KPJ-J1)103,104,54
103     WRITE(6,119)J1,ISEGE
        STOP
104     IF (NPRINT.EQ.0) GO TO 122
        WRITE(6,31)ISEGE,T1
        DO 121 IK=1,NMP(ISEGE)
121     WRITE(6,35)IK,PN(IK)
122     CONTINUEC
C  FORMAT USED
C
 30     FORMAT(/////,5X,'MERIDIONAL DESTRIBUTION OF NORMAL PRESSURE'
     $ ' ON SEGMENT   ',I5,10X,' AT TIME',F10.5,//,10X,
     $ ' MESH POINT NUMBER          NORMAL PRESSURE',/)
 31     FORMAT(/////,5X,'GENERATED PRESSURE ON SEGMENT   ',I5,10X,'AT',
     $ ' TIME',F10.5,//,10X,' MESH POINT NUMBER          NORMAL',
     $' PRESSURE',/)
 35     FORMAT(15X,I4,20X,F8.4)
 119    FORMAT(//,5X,'NODAL POINT CARD ERROR  J1=',I5,/,5X,
     $' SEGMENT NUMBER       I='I5)
C
C
        RETURN
        END
C
C
C*****************
C=DECK= AMPLIT   *
C*****************
C
      SUBROUTINE AMPLIT (J,N,XX)
C  CALCULATES A(J,N), COEFFICIENT OF FOURIER SINE OR COSINE OR
C  MIXED SERIES CORRESPONDING TO GIVEN CIRCUMFERENTIAL WAVENUMBER N.
C  COSINE SERIES IS USED FOR NEGATIVE CIRCUMFERENIAL N WHILE SINE
C  SERIES IS USED FOR POSTIVE N. THIS SUBROUTINE IS THAT CALLED  SUB-
C  ROUTINE FCOEF USED IN BOSOR4 PROGRAM.
C
5512      COMMON XPLUS(100),XMINUS(100),YPLUS(100),YMINUS(100)
      COMMON/FC/A(25,50),ISEGE,NZ,FN(500,50)
      PI = 3.1415927
      FL = XX
      FNPI = ABS(FLOAT(N))*PI
      Q = FNPI/FL
      A(ISEGE,J) = 0.0
      DO 30 I=2,NZ
      I1 = I - 1
      H = XPLUS(I) - XPLUS(I1)
      QX = Q*XPLUS(I)
      QX1 = Q*XPLUS(I1)
      CS = COS(QX)
      CS1 = COS(QX1)
      SS = SIN(QX)
      SS1 = SIN(QX1)
      IF (N) 10,10,20
C     COEFFICIENTS OF COSINE SERIES
```

```
 10     Y = YPLUS(I) + YMINUS(I)
        Y1 = YPLUS(I1) + YMINUS(I1)
        C = (Y - Y1)/H
        IF (N.EQ.0) A(ISEGE,J) = A(ISEGE,J) + 0.25*H*(Y + Y1)/FL
        IF (N.EQ.0) GO TO 30
        A(ISEGE,J) = A(ISEGE,J) + (C*(CS-CS1)/Q+Y*SS-Y1*SS1)/FNPI
        GO TO 30
 C      SINE SERIES COEFFICIENTS
 20     Y = YPLUS(I) - YMINUS(I)
        Y1 = YPLUS(I1) - YMINUS(I1)
        C = (Y - Y1)/H
        A(ISEGE,J) = A(ISEGE,J) + (C*(SS-SS1)/Q - Y*CS + Y1*CS1)/FNPI
 30     CONTINUE
        RETURN
        END
 C
 C******************
 C*DECK= FORCE     *
 C******************
 C
 C
        SUBROUTINE FORCE(N,TT,JK,NPRINT)
 C
 C  THIS SUBROUTINE CALCULATES THE FORCES PER UNIT CIRCUMFERENCE AT
 C  EACH STATION POINTS. THE ROUTINE USED HERE FOLLOWS SUBROUTINE SRHS
 C  IN BOSOR4 PROGRAM. FN(IK,JK) IS THE FORCE AT STATION POINT NO.IK
C  AT TIME POINT NO. JK;WHERE IK=1,NMPT WHILE JK=1,NTT . S  E  EHE MAIN
C  PROGRAM ABOVE FOR THE DEFINITION OF THE OTHER PARAMETERS USED BELOW.
 C
        COMMON/SEGM/R(500),NMP(25),PN(100),NSEG
        COMMON/FC/A(25,50),ISEGE,NZ,FN(500,50)
        COMMON/FF/DS(500),DH(500),GM(250)
        DOUBLE PRECISION DS,DH
 C
        DIMENSION FOR(305)
        NNN=3*NMP(ISEGE)+4
 C   NOTICE: CORRECT THE CALCULATED NODAL FORCE VECTOR USING THE FOLLOWING
 C           1. FOR A ZERO CIRCUMFERENTIAL WAVES MULTIPLY FN BY PI.
 C           2. FOR CIRCUMFERENTIAL WAVES LESS OR GREATER THAN ZERO
 C              MUTIPLY FN BY 2*PI.
 C   THIS CORRECTIONS FOLLOWS THAT DONE IN  SUBROUTINE MODE IN BOSOR4
 C PROGRAM.
 C
        FAC=1.0
        IF (N.EQ.0) FAC=2.0
        DO 102 K=1,NNN
 102    FOR(K)=0.0
        IZZ=ISEGE-1
        IJ=0
        IK=0
        IF(ISEGE.EQ.1) GO TO 52
        DO 51 I1=1,IZZ
  51    IJ=IJ+NMP(I1)
  52    IH=IJ+1
```

```
      IK=IJ
      DO 10 J=1,NMP(ISEGE)
      IJ=IJ+1
      DK=2*DS(IJ)-DH(IJ)
      HMK=DH(IJ)-DK
      PDSK =DK+DS(IJ)
      PDSH =DH(IJ)+DS(IJ)
      W1   =HMK*PDSK/(16.*DH(IJ)*DS(IJ))
      W2   =PDSK*PDSH/(4.*DH(IJ)*DK)
      W3   =-HMK*PDSH/(16.*DK*DS(IJ))
      II   =3*J+1
      IL   =II-3
      IR   =II+3
      IF(J.EQ.1.OR.J.EQ.NMP(ISEGE)) DS(IJ)=DS(IJ)/2.
      FOR(IL) =FOR(IL)+R(IJ)*W1*DS(IJ)*PN(J)*A(ISEGE,JK)
      FOR(II) =FOR(II)+R(IJ)*W2*DS(IJ)*PN(J)*A(ISEGE,JK)
      FOR(IR) =FOR(IR)+R(IJ)*W3*DS(IJ)*PN(J)*A(ISEGE,JK)
      IF(J.EQ.1.OR.J.EQ.NMP(ISEGE)) DS(IJ)=2*DS(IJ)
 10   CONTINUE
      WRITE(6,91)
      IF(NPRINT.EQ.0) GO TO 95
      IF(N.LE.0) GO TO 60
      WRITE(6,30)N,A(ISEGE,JK)
      GO TO 90
 60   WRITE(6,91)
      WRITE(6,50)N,A(ISEGE,JK)
      GO TO 90
 90   WRITE(6,15)ISEGE,TT
 95   IH1=IH+NMP(ISEGE)-1
      DO 100 I=1,NMP(ISEGE)
      IK=IK+1
      JJ=3*I+1
100   FN(IK,JK)=FOR(JJ)*FAC
      IF(NPRINT.EQ.1)WRITE(6,80)(FN(I,JK),I=IH,IH1)
 15   FORMAT(/,5X,' NODAL FORCE VECTOR FOR SEGMENT NUMBER ',I5,5X,
     $' AT TIME ',F10.5,/)
 30   FORMAT(11H WAVE NO.N=I6,32H. CIRC. HARMONIC AMPLITUDE A(N)=1PE15.5.
     $,45H. CIRC. DISTRIBUTION Y(THETA)= A*SIN(N*THETA)     )
 50   FORMAT(11H WAVE NO.N=I6,32H. CIRC. HARMONIC AMPLITUDE A(N)=1PE15.5.
     $,45H. CIRC. DISTRIBUTION Y(THETA)= A*COS(N*THETA)     )
 80   FORMAT(5X,1P10E12.5)
 91   FORMAT(//)
      RETURN
      END
C
C*****************
C*DECK= SOLVE    *
C*****************
C
      SUBROUTINE SOLVE (NPRESS,NPSTRE,NSTRT,NASME,NDTOP,NT1,DT,ROW)
C
C  THIS SUBROUTINE SOLVE THE DIFFERENTIAL EQUATION UTILIZING
C  WILSON-THETA METHOD OUTLINED IN "NUMERICAL METHODS IN FINITE
C  ELEMENT ANALYSIS" BY:K .J. BATHE AND E .L. WILSON,PRENTIC-E
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C  HALL,INC.,ENGLEWOOD CILLS,NEW JERSEY.
C  SEE THE ABOVE REFERENCE PP.319-321 (VALUE OF 1.4 FOR THETA IS
C  USED ).
C
        COMMON/INT/FE(250,100),FR(250),NTT,NMODES,NMPT,T(100),DAMRAT
        COMMON/MEMBRN/N1(500,250),N2(500,250),N12(500,250)
        COMMON/MOMENT/M1(500,250),M2(500,250),M12(500,250)
        COMMON/VIBRAT/NVEC(50),NM,NSTART,NFIN,INCR,NMERD
        COMMON/ASME/SIGM(250,36),F1(250),F2(250),F12(250),STLOC(250,36)
        COMMON/DISP/U(500,250),V(500,250),W(500,250),THETA(36)
        COMMON/SEGM/R(500),NMP(25),PN(100),NSEG
        COMMON/GEOM/SMPA(500),CURV1(500),STA(500)
        DIMENSION B(250),BB(250),X0(250),X1(250),X2(250)
        DIMENSION SIG1(250),SLC1(250),TLC1(250)
        DOUBLE PRECISION X0,X1,X2,B,BB
        CHARACTER*16 FILN
C
C  CALCULATE CONSTANTS REQUIRED FOR WILSON-THETA METHOD
      A0=6.0/(DT**2*1.4**2)
      A1=3.0/DT/1.4
      A2=2*A1
      A3=0.7*DT
      A4=A0/1.4
      A5=-A2/1.4
      A6=1-3.0/1.4
      A7=DT/2.
      A8=DT**2/6.
      PI=3.141592654C
C CHECK FOR RE-START POSSIBILITY
C
C
C     STARTING TIME IS SET EQUAL TO ZERO UNLESS OTHERWISE SPECIFIED BY
C     TH USER
C
      STRTIM =0
      IF (NSTRT.EQ.0) GO TO 100
      OPEN(UNIT=30,STATUS='OLD')
      READ(30,*)STRTIM
      DO 110 I=1,NMODES
 110  READ(30,*)X2(I),X1(I),X0(I)
      DO 111 IK=1,NMERD
      DO 111 IL=1,NMPT
 111  READ(30,*)SIGM(IL,IK),STLOC(IL,IK)
      DO 119 I=1,NMPT
 119  READ(30,*)SIG1(I),SLC1(I),TLC1(I)
      READ(30,*)CLC
      CLOSE(UNIT=30)
      GO TO 105
C
C INITIALIZE LAMDA"X0" ,LAMDA DOT"X1" ,LAMDA DOUBLE DOT2"
100   DO 20 I=1,NMODES
      X0(I)=0
      X1(I)=0
  20  X2(I)=FE(I,1)
```

```
C
C
C INITIALIZE SIGM(I),SLOC(I) ,I=1,NMPT).THESE WILL BE USED IN THE ASME
C LEVEL C CODE CHECK.
C
      DO 205 IK=1,NMERD
      DO 205 I=1,NMPT
      SIGM(I,IK)=0.0
205   STLOC(I,IK)=0
      DO 206 I=1,NMPT
      SIG1(I)=0
      SLC1(I)=0
 206  TLC1(I)=0
      CLC=0
C  STEP BY STEP SOLUTION
C
105   TIME =STRTIM
      DO 25 NNN=1,NT1
C
C  TIME1 = CURRENT TIME+DT. THIS IS USED TO EVALUATE THE R.H.S.
C  IN THE CHARACTERISTIC EQUATION AS SHOWN IN THE AOVE REF. PAGE 320.
C
      TIME1=TIME+DT
C
C FIND THE CURRENT LOCATION ON THE FE-TIME RELATION CURVE USING LINEAR
C INTERPOLATION.
C
C  1: EQUIVALENT FORCE VECTOR AT TIME T IS  B(I) ; I=1,NMODES
C  2: EQUIVALENT FORCE VECTOR AT TIME=T+DT IS BB(I) ; I=1,NMODES
C
      DO 30 I=1,NMODES
      XD=X0(I)
      XV=X1(I)
      XA=X2(I)
      IF (NNN.EQ.1) GO TO 30
      DO 60 JJ=1,2
      TR=TIME
      IF (JJ.EQ.2) TR=TIME1
      DO 50 J=2,NTT
      IF(TR.GT.T(NTT)) TR=T(NTT)
      IF(TR.LT.T(J)) GO TO 55
 50   CONTINUE
 55   DB=(FE(I,J)-FE(I,J-1))/(T(J)-T(J-1))
      F=FE(I,J-1)+DB*(TR-T(J-1))
      IF (JJ.EQ.1) B(I)=F
      IF (JJ.EQ.2) BB(I)=F
 60   CONTINUE
C
C CALCULATE THE EFFECTIVE RIGHT HAND SIDE TO BE USED IN WILSON-THETA
C METHOD (SEE BOTTOM OF PAGE 320 IN THE ABOVE MENTIONED REFERNCE)
C AND CALCULATE X0,X1,X2.
C
      B(I)=B(I)+1.4*(BB(I)-B(I))+A0*XD+A2*XV+2*XA+DAMRAT*(A1*XD+2.*XV+
     $A3*XA)
```

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```
C  CALCULATE XO(I) AT TIME=TIME+THETA*DT ;(THETA=1.4)
      OMG=2*PI*FR(I)
      XO(I)=B(I)/(OMG**2+AO+A1*DAMRAT*OMG)
C
C  CALCULATE XO,X1,X2 AT TIME=T+DT
C
      X2(I)=A4*(XO(I)-XD)+A5*XV+A6*XA
      X1(I)=XV+A7*(X2(I)+XA)
      XO(I)=XD+DT*XV+A8*(X2(I)+2*XA)
30    CONTINUE
      IF(NNN.EQ.1.AND.NSTRT.EQ.1)  GO TO 27
      IF(NNN.EQ.1)   GO TO 26
      IF(NNN.EQ.NT1) GO TO 26
      IF(((NNN-1)/NPSTRE)*NPSTRE.NE.(NNN-1)) GO TO 70
26    J=0
      IF(TIME.GT.T(NTT)) TIME =T(NTT)
      WRITE(6,35)TIME
      NJ=NSTART
      DO 70 KI=1,NM
      DO 80 IK=1,NVEC(KI)
      J=J+1
80    WRITE(6,45)NJ,IK,XO(J)
      NJ=NJ+INCR
70    CONTINUE
      NNST=((NNN-1)/NPSTRE)*NPSTRE
      NNAS=((NNN-1)/NASME)*NASME
      NNPR=((NNN-1)/NPRESS)*NPRESS
      IF(NNN.EQ.1.OR.NNN.EQ.NT1) GO TO 29
      IF(NNST.EQ.(NNN-1).OR.NNAS.EQ.(NNN-1)) GO TO 29
      GO TO 28
29    CALL STRESS (NPSTRE,XO,NMODES,NMPT,NT1,NNN,TIME,NSTRT,NASME,
     $TIME,ROW)
28    IF(NNN.EQ.1.OR.NNN.EQ.NT1.OR.NNPR.EQ.(NNN-1))
     $CALL PRECAL(XO,TIME,NT1,NNN,NSTRT)
27    TIME =TIME+DT
25    CONTINUE
C
C FIND TEH ENVELOP OF THE STRESS INTENSITIES OF THE DIFFERET MERIDIONAL
C LOCATIONS. SIG1(I) IS THE STRESS INTENSITY. SLC1(I) IS THE CIRCUM-
C FERENTIAL LOCATION . TLC1(I) IS THE TIME OF OCCURENCE. I IS THE MESH
C POINT NUMBER.
C
        DO 94 II=1,NMERD
        DO 97 I=1,NMPT
        IF(SIGM(I,II).LT.SIG1(I)) GO TO 97
        SIG1(I)=SIGM(I,II)
        SLC1(I) = THETA(II)
        TLC1(I) =STLOC(I,II)
 97     CONTINUE
 94     CONTINUE
C
       DO 180 KI=1,NMERD
       I=0
       DO 180 IX=1,NSEG
      WRITE(6,155)IX,THETA(KI)
      DO 190 K=1,NMP(IX)
      I=I+1
 190  WRITE(6,175)K,SIGM(I,KI),STLOC(I,KI)
 180  CONTINUE
C
C
C PRINT OUT THE ENVELOP OF THE STRESS INTENSITY OVER THE ENTIRE
C STRUCTURE.
C
      I=0
      DO 95 IX=1,NSEG
      WRITE(6,96)IX
      DO 99 K=1,NMP(IX)
      I=I+1
      IF(SIG1(I).LT.CLC) GO TO 99
      CLC=SIG1(I)
      SLC=SLC1(I)
      TLC=TLC1(I)
      KP=K
      KS=IX
  99  WRITE(6,98)K,SIG1(I),SLC1(I),TLC1(I)
  95  CONTINUE
C
C
C     WRITE MODAL ACCELERATION,VELOCITY AND DISPLACEMENT ON FILE30
C     FOR POSSIBLE RESTART.
C
C
      OPEN(UNIT=30,STATUS='NEW')
      TIME=TIME-DT
      WRITE(30,*)TIME
      DO 101 KJ=1,NMODES
101   WRITE(30,*)X2(KJ),X1(KJ),X0(KJ)
      DO 102 II=1,NMERD
      DO 102 IX=1,NMPT
102   WRITE(30,*)SIGM(IX,II),STLOC(IX,II)
      DO 103 I=1,NMPT
103   WRITE(30,*)SIG1(I),SLC1(I),TLC1(I)
      WRITE(30,*)CLC
      CLOSE(UNIT=30)
C
       WRITE(6,135)
       WRITE(6,136)CLC,TLC,SLC,KP,KS
C
C
 35   FORMAT(//,15X,' MODAL CO-ORDINATE AT TMIE =',1PE12.5,
     $//,1'CIRCUMFERENTIAL WAVE',10X,'MODE NUMBER',
     $10X,'MODAL CO-ORDINATE',//)
 45   FORMAT(16X,I5,22X,I3,15X,1PE14.7,2(5X,1PE14.7))
 96   FORMAT(//,20X,'MAXIMUM ENVELOPE OF THE STRESS INTENSITY',
     $' FOR SEGMENT ',I2,//,11X,'MESH POINT',9X,' STRESS',
     $' INTENSITY',9X,' THETA (DEGREES)',9X,' TIME (SEC.)',//)
 98   FORMAT(10X,0PI6,16X,1PE12.5,17X,0PF7.3,14X,1PE12.5)
 135  FORMAT(/////)
 136  FORMAT(5X,' MAXIMUM MEMBRANE STRESS INTENSITY OVER THE ENTIRE',
     $' STRUCTURE : ',1PE12.5,/,5X,' OCCURED AT TIME',45X,':',1PE12.5,
     $/,5X,' AND AT CIRCUMFERENTIAL LOCATION ( THETA-DEGREES)',12X,':',
     $0PF12.5,/,5X,' AND AT STATION POINT NUMBER :',0PI10,9X,
     $' OF SEGMENT :',0PI12,//)
155   FORMAT(//,20X,'STRESS INTENSITY FOR SEGMENT ',I2,5X,
     $' AT  CIRCUMFERENTIAL ANGLE ',F7.3,//,11X,'MESH POINT',
     $14X,' STRESS INTENSITY',8X,' TIME (SEC.)',//)
175   FORMAT(11X,0PI6,21X,1PE12.5,8X,1PE12.3)
      RETURN
      END
C
C****************
C*DECK= STRESSE *
C****************
C
C
       SUBROUTINE STRESS  (NNST,LAMDA,NMODES,NMPT,NF,NS,TK,NSTRT,
     $ NNAS,TIME,ROW)
C
       COMMON/SEGM/R(500),NMP(25),PN(100),NSEG
       COMMON/VIBRAT/NVEC(50),NM,NSTART,NFIN,INCR,NMERD
       COMMON/MEMBRN/N1(500,250),N2(500,250),N12(500,250)
       COMMON/MOMENT/M1(500,250),M2(500,250),M12(500,250)
       COMMON/DISP/U(500,250),V(500,250),W(500,250),THETA(36)
       COMMON/ASME/SIGM(250,36),F1(250),F2(250),F12(250),STLOC(250,36)
       DIMENSION DU(200),DV(200),DW(200),FM1(200),FM2(200),FM12(200)
       REAL N1,N2,N12,M1,M2,M12
       DOUBLE PRECISION LAMDA(NMODES)
C
C DO NOT PERFORM ANY CALCULATIONS AT TIME =0.0 SINCE LAMDA IS ZERO.
C
C
C
       DO 50 II=1,NMERD
       J=0
       FN1=NSTART
       DO 51 IL=1,NMPT
       DU(IL)=0.0
       DV(IL)=0.0
       DW(IL)=0.0
       F1(IL)=0.0
       F2(IL)=0.0
       F12(IL)=0.0
       FM1(IL)=0.0
       FM2(IL)=0.0
 51    FM12(IL)=0.0
       IF(NS.EQ.1.AND.NSTRT.EQ.0) GO TO 140
C
C CALCULATE THE STRESS RESULTANTS AT THE LAST INTEGRATION STEP
C
       IF(NS.EQ.NF) GO TO 82
C
```

```
C
C  DO NOT CALCULATE U,V,W,M1,M2 AND M12 IF THEY ARE NOT REQUIRED
C  FOR THIS INTEGRATION STEP.
C
C
       IF(NNSTE.NE.(NS-1)) GO TO 81
C
C CALCULATE THE MERIDIONAL DISPLACEMENT "U"
C
 82    DO 60 IS=1,NM
       CALL COMPON (A1,B1,THETA(II),FN1)
       DO 65 JK=1,NVEC(IS)
       J=J+1
       DO 65 I=1,NMPT
 65    DU(I)=DU(I)+U(I,J)*LAMDA(J)*A1
       FN11=FN11+INCR
 60    CONTINUE
       J=0
       FN1=NSTART
C
C CALCULATE THE CIRCUMFERENTIAL DISPLACEMENT "V"
C
       DO 70 IS=1,NM
       CALL COMPON (A1,B1,THETA(II),FN1)
       DO 75 JK=1,NVEC(IS)
       J=J+1
       DO 75 I=1,NMPT
 75    DV(I)=DV(I)+V(I,J)*LAMDA(J)*B1
       FN1=FN1+INCR
 70    CONTINUE
       J=0
       FN1=NSTART
C
C CALCULATE THE NORMAL DISPLACEMENT "W"
C
       DO 80 IS=1,NM
       CALL COMPON (A1,B1,THETA(II),FN1)
       DO 85 JK=1,NVEC(IS)
       J=J+1
       DO 85 I=1,NMPT
 85    DW(I)=DW(I)+W(I,J)*LAMDA(J)*A1
       FN1=FN1+INCR
 80    CONTINUE
       J=0
       FN1=NSTART
C
C CALCULATE THE MERIDIONAL STRESS RESULTANTS "N1"
C
 81    DO 90 IS=1,NM
       CALL COMPON (A1,B1,THETA(II),FN1)
       DO 95 JK=1,NVEC(IS)
       J=J+1
       DO 95 I=1,NMPT
 95    F1(I)=F1(I)+N1(I,J)*LAMDA(J)*A1
```

```
      FN1=FN1+INCR
 90   CONTINUE
      J=0
      FN1=NSTART
C
C CALCULATE THE CIRCUMFERENTIAL STRESS RESULTANTS "N2"
C
      DO 100 IS=1,NM
      CALL COMPON (A1,B1,THETA(II),FN1)
      DO 105 JK=1,NVEC(IS)
      J=J+1
      DO 105 I=1,NMPT
 105  F2(I)=F2(I)+N2(I,J)*LAMDA(J)*A1
      FN1=FN1+INCR
 100  CONTINUE
      J=0
      FNSTART
C
C CALCULATE THE SHEAR STRESS RESULTANTS "N12"
C
      DO 110 IS=1,NM
      CALL COMPON (A1,B1,THETA(II),FN1)
      DO 115 JK=1,NVEC(IS)
      J=J+1
      DO 115 I=1,NMPT
 115  F12(I)=F12(I)+N12(I,J)*LAMDA(J)*B1
      FN1=FN1+INCR
 110  CONTINUE
C
C  UPDATE THE STRESS INTENSITY ENVELOP AT THE LAST INTEGRATION STEP.
C
      IF (NS.EQ.NF) GO TO 132
C
C
C DO NOT CALCULTE THE ASME STRESS INTENSITIES IF THEY ARE NOT REQUIRED AT
C  AT THE CURRENT INTEGRATION STEP. THIS IS CONTROLED BY NASME.
C
      IF (NNAS.NE.(NS-1)) GO TO 131
 132  CALL ASME (NMPT,TIME,II,ROW)
C
C CALCULATE THE STRESS RESULTANTS AT THE LAST INTEGRATION STEP.
C
      IF (NS.EQ.NF) GO TO 131
C
C DO NOT CALCULATE THE BENDING STRESS RESULTANTS IF THEY ARE NOT NEEDED
C AT THIS INTEGRATION STEP. THIS IS CONTROLED BY NPSTRE.
C
      IF (NNST.NE.(NS-1)) GO TO 171
C
C
 131  J=0
      FN1=NSTART
C
C CALCULATE THE MERIDIONAL MOMENT STRESS RESULTANTS "M1"
```

```
C
      DO 120 IS=1,NM
      CALL COMPON (A1,B1,THETA(II),FN1)
      DO 125 JK=1,NVEC(IS)
      J=J+1
      DO 125 I=1,NMPT
 125  FM1(I)=FM1(I)+M1(I,J)*LAMDA(J)
      FN1=FN1+INCR
 120  CONTINUE
      J=0
      FN1=NSTART
C
C CALCULTAE THE CIRCUMFERENTIAL MOMENT STRESS RESULTANTS "M2"
C
      DO 130 IS=1,NM
      CALL COMPON (A1,B1,THETA(II),FN1)
      DO 135 JK=1,NVEC(IS)
      J=J+1
      DO 135 I=1,NMPT
 135  FM2(I)=FM2(I)+M2(I,J)*LAMDA(J)*A1
      FN1=FN1+INCR
 130  CONTINUE
      J=0
      FN1=NSTART
C
C CALCULATE THE TWISTING MOMENT STRESS RESULTANTS "M12"
C
      DO 140 IS=1,NM
      CALL COMPON (A1,B1,THETA(II),FN1)
      DO 145 JK=1,NVEC(IS)
      J=J+1
      DO 145 I=1,NMPT
 145  FM12(I)=FM12(I)+M12(I,J)*LAMDA(J)*B1
      FN1=FN1+INCR
 140  CONTINUE
C PRINT OUT THE STRESS RESULTANTS AT THE SPECIFIED MERIDIONAL LOCATIONS.
      I=0
      DO 180 IX=1,NSEG
      WRITE(6,155)IX,TK,THETA(II)
      DO 190 K=1,NMP(IX)
      I=I+1
 190  WRITE(6,175)K,DU(I),DV(I),DW(I),F1(I),F2(I),F12(I),FM1(I),
     $ FM2(I),FM12(I)
 180  CONTINUE
  50  CONTINUE
C
C
C   FORMAT USED TO PRINT OUT THE STRESS RESULTANTS.
C
C
155     FORMAT(//,20X,'STRESS RESULTANTS FOR SEGMENT ',I2,5X,
     $ ' AT TIME ',1PE12.5,' AND CIRCUMFERENTIAL ANGLE ',1PE12.5,
     $ /,2X,'MESH',/,'  POINT',9X,'U',12X,'V',13X,'W',11X,'N1',
     $ 10X,'N2',12X,'N12',10X,'M1',11X,'M2',10X,'M12')
  175   FORMAT(I5,7X,1PE10.3,8(3X,1PE10.3))
  171   RETURN
        END
C
C*****************
C*DECK= COMPON  *
C*****************
C
C
        SUBROUTINE COMPON (A1,B1,THE1,FNK)
C
        PI=3.141592654
        THE=ABS(FNK*(THE1*PI/180.))
C
C  CIRCUMFERENTIAL VARIATION OF  DISPLACEMENT,MEMBRANE AND MOMENT
C  STRESS RESULTANTS.
C
C  1. FOR POSTIVE CIRCUMFERENTIAL WANE NUMBERS.
C
        CO  =COS(THE)
        SI  =SIN(THE)
        IF (FNK.GT.0) GO TO 15
C
C 2. FOR NEGATIVE OR ZERO CIRCUMFERENTIAL WAVE NUMBERS.
C
        SI  =CO
        CO  =SIN(THE)
  15    A1 =SI
        B1 =CO
        IF (FNK.EQ.0) B1 =SI
        RETURN
        END
C
C
C******************
C* DECK= ASME     *
C******************
C
         SUBROUTINE ASME (NMPT,TIME,JK,ROW)
C
C    THIS SUBROUTINE CALCULATES THE ASME STRESS INTENSITY ENVELOP AT
C    ALL NMERD SPECIFIED ON THE CONTROL INFORMATION LINE.
C
       COMMON/GEOM/SMPA(500),CURV1(500),STA(500)
       COMMON/ASME/SIGM(250,36),F1(250),F2(250),F12(250),STLOC(250,36)
C
       DO 10 I=1,NMPT
       TERM1=F1(I)+F2(I)
       TERM2=(F1(I)-F2(I))**2+4*F12(I)**2
       TERM3=SQRT(TERM2)
       THCK=SMPA(I)/ROW
       SIG1=(TERM1+TERM3)/THCK/2.
       SIG2=(TERM1-TERM3)/THCK/2.
       ASIG1=ABS(SIG1)
```

```
      ASIG2=ABS(SIG2)
      ASIG3=ABS(SIG1-SIG2)
      SIGMA=AMAX1(ASIG1,ASIG2,ASIG3)
      IF(SIGMA.LT.SIGM(I,JK))  GO TO 10
      SIGM(I,JK)=SIGMA
      STLOC(I,JK)=TIME
 10   CONTINUE
      RETURN
      END
C
C
C*********************
C*DECK= PRECAL       *
C*********************
C
      SUBROUTINE PRECAL (LAMDA,TI,NK1,NK2,NSTRT)
C
C
C THIS SUBROUTINE CALCULATES AND WRITES THE EQUIVALENT AXISYMMETRIC
C PRESSURES AT A GIVEN CIRCUMFERENTIAL LOCATION. THESE PRESSURES ARE
C USED AS AN INPUT FOR BOSOR5 ANALYSIS.
C
C
      COMMON/SEGM/R(500),NMP(25),PN(100),NSEG
      COMMON/INT/FE(250,100),FR(250),NTT,NMODES,NMPT,T(100),DAMRAT
      COMMON/MEMBRN/N1(500,250),N2(500,250),N12(500,250)
      COMMON/MOMENT/M1(500,250),M2(500,250),M12(500,250)
      COMMON/DISP/U(500,250),V(500,250),W(500,250),THETA(36)
      COMMON/VIBRAT/NVEC(50),NM,NSTART,NFIN,INCR,NMERD
      COMMON/GEOM/SMPA(500),CURV1(500),STA(500)
      REAL N1,N2,N12,M1,M2,M12
      DOUBLE PRECISION LAMDA(NMODES)
      CHARACTER*16 FILN
C
C
      PI=3.141592654
      DO 50 II=1,NMERD
      J=0
C
C ***   WRITE THE CALCULATED EQUIVALENT AXISYMMETRIC PRESSURES AT
C *** EVERY MERIDIONAL ON SEPARATE FILE CALLED PRESSR.DAT;1..
C *** PRESSR.DAT;2.......PRESSR.DAT;NMERD.
C
C THE FOLLOWINGS ARE TO DEFINE WHICH FILE OF FOR015.DAT;? TO BE
C USED AT THE CURRENT TIME AND MERDIAN.
C
      K=II/10+48
             LK=II-II/10+48
      FILN='PRESSR.DAT;'//CHAR(K)//CHAR(LK)
C
C*** FIRST OPEN NEW UNITS OF FOR015.DAT FOR EVERY MERDIONAL.
C
C
C*** USE THE ALREADY CREATED FILES FROM THE PREVIOUS RUN IF THIS IS
```

```
C    A RESTART CASE.
C
      IF(NSTRT.EQ.1) GO TO 10
      IF(NK2.EQ.1)OPEN(UNIT=15,STATUS='NEW',FILE=FILN)
C
C*** WRITE THE CALCULATED PRESSURES OF THE THE FOLLOWING TIME STEPS
C*** ON THE SAME FILE FOR EACH MERDIONAL.
C
10    OPEN(UNIT=15,ACCESS='APPEND',STATUS='OLD',FILE=FILN)
      WRITE(15,*)TI
      DO 60 IX=1,NSEG
      DO 60 K=1,NMP(IX)
      J=J+1
      FN1=NSTART
C
C PTA IS THE TOTAL TANGENTIAL AXISYMMETRIC PRESSURE AT MESH POINT K IN
C THE CURRENT SEGMENT IX .
C
C PNA IS THE TOTAL AXISYMMETRIC NORMAL PRESSURE AT MESH POINT K IN
C THE CURRENT SEGMENT IX .
      PTA=0.0
      PNA=0.0
      I =0
      IF(NK2.EQ.1) GO TO 101
C
C CALCULATE THE PRESSURES AT A GIVEG MERIDIAN.
C
      DO 70 IS=1,NM
      THE=ABS(FN1*(THETA(II)*PI/180.))
C
C  CIRCUMFERENTIAL VARIATION OF  PRESSURES.
C
C  1. FOR POSTIVE CIRCUMFERENTIAL WAVE NUMBERS.
C
      CO  =COS(THE)
      SI  =SIN(THE)
      C   =SI
C
C 2. FOR NEGATIVE OR ZERO CIRCUMFERENTIAL WAVE NUMBERS.
C
      IF(FN1.LE.0) C=CO
C
      DO 80 JK=1,NVEC(IS)
      I=I+1
C
C TERM1*U AND TERM1*W REPRESENT THE INTERTIAL PRESSURES OR BY
C D'ALEMBERTS' PRINCIPLE THE NEGATIVE OF EQUIVALENT STATIC PRESSURES.
C
      TERM1=((FR(I)*2.0*PI)**2)*SMPA(J)
      IF (R(J).NE.0) GOTO 148
C
C TERM2 AND TERM4 ARE THOSE CONTAIN THE STERSS RESULTANTS IN THE
C DIFFERENTIAL EQUATION AN FOR AXISYMMETRIC SHELL UNDER NONSYMMETRIC
C LOADS.
```

```
C
      TERM2=0.0
      TERM4=0.0
      GO TO 149
C
C THE FOLLOWING IF STATMENTS ARE USED TO AVOID USING THE STRESS
C RESULTANTS CALCULATED AT THE ADDTIONAL MESH POINT CREATED AUTOMATICALLY
C BY BOSOR4 PROGRAM. SEE THE NOTE GIVEN AT THE BEGING OF THIS PROGRAM.
C
  148 TERM2=FN1*(N12(J,I)/R(J)+M12(J,I)*CURV1(J)/R(J))
      IF(K.EQ.1)TERM3=(R(J+2)*M12(J+2,I)-R(J)*M12(J,I))/
     $ (STA(J+2)-STA(J))
      IF(K.EQ.3)TERM3=(R(J+1)*M12(J+1,I)-R(J)*M12(J-2,I))/
     $ (STA(J+1)-STA(J-2))
      IF(K.EQ.NMP(IX))TERM3=(R(J)*M12(J,I)-R(J-2)*M12(J-2,I))/
     $ (STA(J)-STA(J-2))
      IF(K.EQ.((NMP(IX)-2)))TERM3=(R(J+2)*M12(J+2,I)-R(J-1)*
     $ M12(J-1,I))/(STA(J+2)-STA(J-1))
      TERM3=(R(J+1)*M12(J+1,I)-R(J-1)*M12(J-1,I))/
     $ (STA(J+1)-STA(J-1))
      TERM4=(FN1*M2(J,I)-2*TERM3)*FN1/(R(J)**2)
C
C CALCULATE THE MODAL PRESSURES IN THE MERIDIONAL DIRECTION (PRESST) AND
C IN THE NORMAL DIRECTION (PRESSN).
C
  149 PRESST=TERM1*U(J,I)-TERM2
      PRESSN=TERM1*W(J,I)+TERM4
C
C SUPERIMPOSE THE MODAL PRESSURES TO CALCULATE THE TOTAL AXISYMMETIC
C PRESSURES AT SPECIFIC MERIDIONAL LOCATION (THETA).
C
      PTA=PTA+PRESST*LAMDA(I)*C
 80   PNA=PNA+PRESSN*LAMDA(I)*C
      FN1=FN1+INCR
 70   CONTINUE
C
C WRITE THESE AXISYMMETRIC  PRESSURES FOR THE SPECIFIED MERIDIONAL
C LOCATIONS (THETA(II),I=1,NMERD) ON FILES FOR BOSOR5 ANALYSIS.
C
101   WRITE(15,*)PNA,PTA
 60   CONTINUE
      CLOSE (UNIT=15)
 50   CONTINUE
      RETURN
      END
```

Table A.14 Input Data to LOADS Program for the Containment in Figure 5.6

```
*** FOR DEFINETIONS OF THE LISTED VARIABLES REFER TO SEC A.3.2 ***
*** READ IN CONTROL INFORMATIONS ***
4,0,21,21,11,1,0,1,
*** READ IN THETA(II),II=1,NMERD ***
0,45,90,180
*** READ IN DT AND NT1 SINCE NDTOP IS ONE ON THE FIRST LINE ***
0.0015,667
*** READ IN NSEGL AND NTT ***
3,6
*** READ IN THE FOLLWINGS FOR NSEGL NTT TIMES ***
*** READ IN TIME T ***
0.0
*** READ IN ISEGEL AND NX AT TIME T ***
1,2
*** READ IN CIRCUMFERENTIAL PRESSURE VARIATION AT TIME T  ***
0,0,-0,0
180,0,-180,0
*** READ IN MERIDIONAL PRESSURE VARIATION AT TIME T ***
1,0
33,0
*** READ IN ISEGEL AND NX AT TIME T ***
2,2
*** READ IN CIRCUMFERENTIAL PRESSURE VARIATION AT TIME T  ***
0,0,-0,0
180,0,-180,0
*** READ IN MERIDIONAL PRESSURE VARIATION AT TIME T ***
1,0
61,0
*** READ IN ISEGEL AND NX AT TIME T ***
3,2
*** READ IN CIRCUMFERENTIAL PRESSURE VARIATION AT TIME T  ***
0,0,-0,0
180,0,-180,0
*** READ IN MERIDIONAL PRESSURE VARIATION AT TIME T ***
1,0
59,0
*** READ IN TIME T ***
0.3
*** READ IN ISEGEL AND NX AT TIME T ***
1,4
*** READ IN CIRCUMFERENTIAL PRESSURE VARIATION AT TIME T  ***
0,1,-0,1
45,1,-45,1
45.1,.4,-45.1,.4
180,.4,-180,.4
*** READ IN MERIDIONAL PRESSURE VARIATION AT TIME T ***
1,1
33,1
*** READ IN ISEGEL AND NX AT TIME T ***
2,4
```

```
*** READ IN CIRCUMFERENTIAL PRESSURE VARIATION AT TIME T ***
0,.4,-0,.4
45,.4,-45,.4
45.1,.28,-45.1,.28
180,.28,-180,.28
*** READ IN MERIDIONAL PRESSURE VARIATION AT TIME T ***
1,1
61,1
*** READ IN ISEGEL AND NX AT TIME T ***
3,2
*** READ IN CIRCUMFERENTIAL PRESSURE VARIATION AT TIME T  ***
0,.2,-0,.2
180,.2,-180,.2
*** READ IN MERIDIONAL PRESSURE VARIATION AT TIME T ***
1,1
59,1
*** READ IN TIME T ***
0.6
*** READ IN ISEGEL AND NX AT TIME T ***
1,2
*** READ IN CIRCUMFERENTIAL PRESSURE VARIATION AT TIME T ***
0,.8,-0,.8
180,.8,-180,.8
*** READ IN MERIDIONAL PRESSURE VARIATION AT TIME T ***
1,1
33,1
*** READ IN ISEGEL AND NX AT TIME T ***
2,4
*** READ IN CIRCUMFERENTIAL PRESSURE VARIATION AT TIME T  ***
0,.8,-0,.8
45,.8,-45,.8
45.1,.56,-45.1,.56
180,.56,-180,.56
*** READ IN MERIDIONAL PRESSURE VARIATION AT TIME T ***
1,1
61,1
*** READ IN ISEGEL AND NX AT TIME T ***
3,2
*** READ IN CIRCUMFERENTIAL PRESSURE VARIATION AT TIME T  ***
0,.4,-0,.4
180,.4,-180,.4
*** READ IN MERIDIONAL PRESSURE VARIATION AT TIME T ***
1,1
59,1
*** READ IN TIME T ***
0.75
*** READ IN ISEGEL AND NX AT TIME T ***
1,2
*** READ IN CIRCUMFERENTIAL PRESSURE VARIATION AT TIME T  ***
0,.75,-0,.75
180,.75,-180,.75
*** READ IN MERIDIONAL PRESSURE VARIATION AT TIME T ***
1,1
33,1
```

```
*** READ IN ISEGEL AND NX AT TIME T ***
2,2
*** READ IN CIRCUMFERENTIAL PRESSURE VARIATION AT TIME T  ***
0,.75,-0,.75
180,.75,-180,.75
*** READ IN MERIDIONAL PRESSURE VARIATION AT TIME T ***
1,1
61,1
*** READ IN ISEGEL AND NX AT TIME T ***
3,2
*** READ IN CIRCUMFERENTIAL PRESSURE VARIATION AT TIME T  ***
0,.5,-0,.5
180,.5,-180,.5
*** READ IN MERIDIONAL PRESSURE VARIATION AT TIME T ***
1,1
59,1
*** READ IN TIME T ***
0.9
*** READ IN ISEGEL AND NX AT TIME T ***
1,2
*** READ IN CIRCUMFERENTIAL PRESSURE VARIATION AT TIME T  ***
0,.6,-0,.6
180,.6,-180,.6
*** READ IN MERIDIONAL PRESSURE VARIATION AT TIME T ***
1,1
33,1
*** READ IN ISEGEL AND NX AT TIME T ***
2,2
*** READ IN CIRCUMFERENTIAL PRESSURE VARIATION AT TIME T  ***
0,.6,-0,.6
180,.6,-180,.6
*** READ IN MERIDIONAL PRESSURE VARIATION AT TIME T ***
1,1
61,1
*** READ IN ISEGEL AND NX AT TIME T ***
3,2
*** READ IN CIRCUMFERENTIAL PRESSURE VARIATION AT TIME T  ***
0,.6,-0,.6
180,.6,-180,.6
*** READ IN MERIDIONAL PRESSURE VARIATION AT TIME T ***
1,1
59,1
*** READ IN TIME T ***
.999
*** READ IN ISEGEL AND NX AT TIME T ***
1,2
*** READ IN CIRCUMFERENTIAL PRESSURE VARIATION AT TIME T  ***
0,.6,-0,.6
180,.6,-180,.6
1,1
*** READ IN MERIDIONAL PRESSURE VARIATION AT TIME T ***
33,1
*** READ IN ISEGEL AND NX AT TIME T ***
2,2
```

```
*** READ IN CIRCUMFERENTIAL PRESSURE VARIATION AT TIME T  ***
0,.6,-0,.6
180,.6,-180,.6
*** READ II  S  E  G  E  L AND NX AT TIME T ***
1,1
61,1
*** READ IN ISEGEL AND NX AT TIME T ***
3,2
0,.6,-0,.6
*** READ IN CIRCUMFERENTIAL PRESSURE VARIATION AT TIME T  ***
180,.6,-180,.6
*** READ IN MERIDIONAL PRESSURE VARIATION AT TIME T ***
1,1
59,1
```

Table A.15 Input Data for Interactive VAX Version of B5AMES for Static Analysis of the Containment in Figure 5.6

```
       3 $ NSEG  = number of shell segments (less than 25)
      33 $ NMESH=no. of node points (5=min.;98=max.)SEGMENT NO.( 1)
       3 $ NTYPEH= control integer (1 or 2 or 3) for nodal point spacing
       1 $ NSHAPE= indicator (1,2 or 4) for geometry of meridian
 600.7500$ R1     = radius at beginning of segment (see p. P7)
       0 $ Z1     = axial coordinate at beginning of segment
 600.7500$ R2     = radius at end of segment
     480 $ Z2     = axial coordinate at end of segment
       1 $ IMP   = indicator for imperfection (0=none, 1=some)
       2 $ ITYPE = indicator (1 or 2) for type of imperfection
 1.500000$ WO    = amplitude of pure sinusoidal imperfection
      60 $ WLNGTH= half-wavelength of pure sinusoidal imperfection
       3 $ NTYPEZ= control (1 or 3) for reference surface location
0.7500000$ ZVAL  = distance from leftmost surf. to reference surf.
   Y     $ Do you want to print out r(s), r'(s), etc. for this segment?
       3 $ NRINGS= number (max=20) of discrete rings in this segment
       2 $ NTYPE = control for identification of ring location (2=z, 3=r)
     120 $ Z(I)  = axial coordinate of Ith ring, z( 1)
     240 $ Z(I)  = axial coordinate of Ith ring, z( 2)
     360 $ Z(I)  = axial coordinate of Ith ring, z( 3)
       1 $ NTYPER= type (0 or 1 ) of discrete ring no.( 1)
       1 $ NTYPER= type (0 or 1 ) of discrete ring no.( 2)
       1 $ NTYPER= type (0 or 1 ) of discrete ring no.( 3)
       1 $ NPARTS = number of segments in discrete ring no.( 1)
       1 $ NGEOM = geometry type of ring segment no.( 1)
       1 $ NTEMP = type of temperature distribution in ring seg.( 1)
       1 $ NMATL = type of material for ring segment no.( 1)
       5 $ INTEG = number of integration points for ring seg.( 1)
       0 $ NCREEP = control for creep of ring segment no.( 1)
       0 $ NUMBT = number of temperature distributions in seg.( 1)
   N     $ Are D1,D2,PHI,T,FL for this ring seg. same as for a previous?
0.7500000$ D1 = radial distance to beginning of ring segment( 1)
       0 $ D2 = axial distance to beginning of ring segment( 1)
       0 $ PHI = angle in degrees of ring segment( 1)
       1 $ T = thickness of ring segment( 1)
      36 $ FL = length of ring segment( 1)
   Y     $ Are TEMP1,TEMP2,TEMP3 identical to those of a previous seg.?
   N     $ Is this ring mat'l same as a previously specified ring mat'l?
29000000 $ E = elastic modulus of ring segment no.( 1)
       0 $ ALPHA = thermal expansion coef. of ring segment no.( 1)
   Y     $ Can the material of this ring segment go plastic?
       3 $ NPOINT = no. of points used for s-s curve of ring seg.( 1)
       0 $ EPSILON = strain coord. of the s-s curve, EPS( 1)
0.1724140E-02 $ EPSILON = strain coord. of the s-s curve, EPS( 2)
0.1724140E-01 $ EPSILON = strain coord. of the s-s curve, EPS( 3)
       0 $ SIGMA   = stress coord. of the s-s curve, SIG( 1)
   50000 $ SIGMA   = stress coord. of the s-s curve, SIG( 2)
   50000 $ SIGMA   = stress coord. of the s-s curve, SIG( 3)
 18.75000$ XS = radial coordinate of shear center, ring no.( 1)
       0 $ YS = axial coordinate of shear center, ring no.( 1)
```

```
        1 $ NPARTS = number of segments in discrete ring no.( 2)
        1 $ NGEOM = geometry type of ring segment no.( 1)
        1 $ NTEMP = type of temperature distribution in ring seg.( 1)
        1 $ NMATL = type of material for ring segment no.( 1)
        5 $ INTEG = number of integration points for ring seg.( 1)
        0 $ NCREEP = control for creep of ring segment no.( 1)
        0 $ NUMBT = number of temperature distributions in seg.( 1)
     Y    $ Are D1,D2,PHI,T,FL for this ring seg. same as for a previous?
     Y    $ Are TEMP1,TEMP2,TEMP3 identical to those of a previous seg.?
     Y    $ Is this ring mat'l same as a previously specified ring mat'l?
 18.75000$ XS = radial coordinate of shear center, ring no.( 2)
        0 $ YS = axial coordinate of shear center, ring no.( 2)
        1 $ NPARTS = number of segments in discrete ring no.( 3)
        1 $ NGEOM = geometry type of ring segment no.( 1)
        1 $ NTEMP = type of temperature distribution in ring seg.( 1)
        1 $ NMATL = type of material for ring segment no.( 1)
        5 $ INTEG = number of integration points for ring seg.( 1)
        0 $ NCREEP = control for creep of ring segment no.( 1)
        0 $ NUMBT = number of temperature distributions in seg.( 1)
     Y    $ Are D1,D2,PHI,T,FL for this ring seg. same as for a previous?
     Y    $ Are TEMP1,TEMP2,TEMP3 identical to those of a previous seg.?
     Y    $ Is this ring mat'l same as a previously specified ring mat'l?
 18.75000$ XS = radial coordinate of shear center, ring no.( 3)
        0 $ YS = axial coordinate of shear center, ring no.( 3)
      N    $ Do you want generaliormation on loading?
          0 $ NTSTAT = number of temperature callout points along meridian
          1 $ NPSTAT = number of meridional callouts for pressure
          0 $ PN = normal pressure, positive as shown on p. P41.
          0 $ PT = meridional traction, positive along increasing s.
          1 $ ISTEP = control integer for time variation of pressure
      N    $ Do you want to print out distributed loads along meridian?
          0 $ LINTYP=control for line loads or disp.(0=none,1=some)
      N    $ Do you want to include smeared stiffeners?
          1 $ LAYERS = number of layers (max. = 6)
      N    $ Are all the layers of constant thickness?
          1 $ MATL = type of material for shell wall layer no.( 1)
   11153846 $ G(i) = shear modulus of ith layer, G( 1)
   29000000 $ EX(i)= modulus in meridional direction, EX( 1)
   29000000 $ EY(i)= modulus in circumferential direction, EY( 1)
  0.3000000$ UXY(i)= Poisson's ratio (EY*UXY = EX*UYX).   UXY( 1)
          0 $ ALPHA1(i)=coef. thermal exp. in merid. direction, ALPHA1( 1)
          0 $ ALPHA2(i)=coef. thermal exp. in circ. direction, ALPHA2( 1)
      Y    $ Do you wish to include plasticity in this segment?
      N    $ Do you wish to include creep in this segment?
      Y    $ Is this a new shell wall material?
          3 $ NPOINT = number of points in s.s.curve, layer no.( 1)
          5 $ NITEG=no. integration pts. thru thickness, layer no.( 1)
      N    $ Do you want to use power law for stress-strain curve?
          0 $ EPS(i)=strain coordinates of s-s curve, EPS( 1)
 0.1724140E-02 $ EPS(i)=strain coordinates of s-s curve, EPS( 2)
 0.1724140E-01 $ EPS(i)=strain coordinates of s-s curve, EPS( 3)
          0 $ SIG(i)=stress coordinates of s-s curve, SIG( 1)
      50000 $ SIG(i)=stress coordinates of s-s curve, SIG( 2)
      50000 $ SIG(i)=stress coordinates of s-s curve, SIG( 3)
```

```
        3 $ NTIN = number of meridional callouts for variable thickness
        2 $ NTYPE = control for meaning of thickness callout (2=z, 3=r)
        0 $ Z(I)  = axial coordinate of Ith thickness callout, z( 1)
      465 $ Z(I)  = axial coordinate of Ith thickness callout, z( 2)
      480 $ Z(I)  = axial coordinate of Ith thickness callout, z( 3)
 1.500000$ TIN(i) = thickness at Ith callout, TIN( 1)
 1.500000$ TIN(i) = thickness at Ith callout, TIN( 2)
 1.000000$ TIN(i) = thickness at Ith callout, TIN( 3)
    N     $ Do you want to have C(i,j) printed for this segment?
       61 $ NMESH=no. of node points (5=min.;98=max.)SEGMENT NO.( 2)
        3 $ NTYPEH= control integer (1 or 2 or 3) for nodal point spacing
        1 $ NSHAPE= indicator (1,2 or 4) for geometry of meridian
 600.7500$ R1      = radius at beginning of segment (see p. P7)
      480 $ Z1      = axial coordinate at beginning of segment
 600.7500$ R2      = radius at end of segment
     1200 $ Z2      = axial coordinate at end of segment
        1 $ IMP    = indicator for imperfection (0=none, 1=some)
        2 $ ITYPE = indicator (1 or 2) for type of imperfection
 1.500000$ WO     = amplitude of pure sinusoidal imperfection
      120 $ WLNGTH= half-wavelength of pure sinusoidal imperfection
        3 $ NTYPEZ= control (1 or 3) for reference surface location
0.7500000$ ZVAL  = distance from leftmost surf. to reference surf.
    Y     $ Do you want to print out r(s), r'(s), etc. for this segment?
        3 $ NRINGS= number (max=20) of discrete rings in this segment
        2 $ NTYPE = control for identification of ring location (2=z, 3=r)
      480 $ Z(I)  = axial coordinate of Ith ring, z( 1)
      720 $ Z(I)  = axial coordinate of Ith ring, z( 2)
      960 $ Z(I)  = axial coordinate of Ith ring, z( 3)
        1 $ NTYPER= type (0 or 1 ) of discrete ring no.( 1)
        1 $ NTYPER= type (0 or 1 ) of discrete ring no.( 2)
        1 $ NTYPER= type (0 or 1 ) of discrete ring no.( 3)
        1 $ NPARTS = number of segments in discrete ring no.( 1)
        2 $ NGEOM = geometry type of ring segment no.( 1)
        2 $ NTEMP = type of temperature distribution in ring seg.( 1)
        2 $ NMATL = type of material for ring segment no.( 1)
        5 $ INTEG = number of integration points for ring seg.( 1)
        0 $ NCREEP = control for creep of ring segment no.( 1)
        0 $ NUMBT = number of temperature distributions in seg.( 1)
    N     $ Are D1,D2,PHI,T,FL for this ring seg. same as for a previous?
0.2500000$ D1 = radial distance to beginning of ring segment( 1)
        0 $ D2 = axial distance to beginning of ring segment( 1)
        0 $ PHI = angle in degrees of ring segment( 1)
        1 $ T = thickness of ring segment( 1)
       36 $ FL = length of ring segme(1)
    Y     $ Are TEMP1,TEMP2,TEMP3 identical to those of a previous seg.?
    N     $ Is this ring mat'l same as a previously specified ring mat'l?
 29000000 $ E = elastic modulus of ring segment no.( 1)
        0 $ ALPHA = thermal expansion coef. of ring segment no.( 1)
    Y     $ Can the material of this ring segment go plastic?
        3 $ NPOINT = no. of points used for s-s curve of ring seg.( 1)
        0 $ EPSILON = strain coord. of the s-s curve, EPS( 1)
0.1741400E-02 $ EPSILON = strain coord. of the s-s curve, EPS( 2)
0.1741400E-01 $ EPSILON = strain coord. of the s-s curve, EPS( 3)
        0 $ SIGMA   = stress coord. of the s-s curve, SIG( 1)
     50000 $ SIGMA   = stress coord. of the s-s curve, SIG( 2)
     50000 $ SIGMA   = stress coord. of the s-s curve, SIG( 3)
  18.25000$ XS = radial coordinate of shear center, ring no.( 1)
         0 $ YS = axial coordinate of shear center, ring no.( 1)
         1 $ NPARTS = number of segments in discrete ring no.( 2)
         2 $ NGEOM = geometry type of ring segment no.( 1)
         0 $ NTEMP = type of temperature distribution in ring seg.( 1)
         2 $ NMATL = type of material for ring segment no.( 1)
         5 $ INTEG = number of integration points for ring seg.( 1)
         0 $ NCREEP = control for creep of ring segment no.( 1)
         0 $ NUMBT = number of temperature distributions in seg.( 1)
    Y      $ Are D1,D2,PHI,T,FL for this ring seg. same as for a previous?
    Y      $ Are TEMP1,TEMP2,TEMP3 identical to those of a previous seg.?
    Y      $ Is this ring mat'l same as a previously specified ring mat'l?
  18.25000$ XS = radial coordinate of shear center, ring no.( 2)
         0 $ YS = axial coordinate of shear center, ring no.( 2)
         1 $ NPARTS = number of segments in discrete ring no.( 3)
         2 $ NGEOM = geometry type of ring segment no.( 1)
         2 $ NTEMP = type of temperature distribution in ring seg.( 1)
         2 $ NMATL = type of material for ring segment no.( 1)
         5 $ INTEG = number of integration points for ring seg.( 1)
         0 $ NCREEP = control for creep of ring segment no.( 1)
         0 $ NUMBT = number of temperature distributions in seg.( 1)
    Y      $ Are D1,D2,PHI,T,FL for this ring seg. same as for a previous?
    Y      $ Are TEMP1,TEMP2,TEMP3 identical to those of a previous seg.?
    Y      $ Is this ring mat'l same as a previously specified ring mat'l?
  18.25000$ XS = radial coordinate of shear center, ring no.( 3)
         0 $ YS = axial coordinate of shear center, ring no.( 3)
    N      $ Do you want general information on loading?
         0 $ NTSTAT = number of temperature callout points along meridian
         1 $ NPSTAT = number of meridional callouts for pressure
         0 $ PN = normal pressure, positive as shown on p. P41.
         0 $ PT = meridional traction, positive along increasing s.
         1 $ ISTEP = control integer for time variation of pressure
    N      $ Do you want to print out distributed loads along meridian?
         0 $ LINTYP=control for line loads or disp.(0=none,1=some)
    N      $ Do you want to include smeared stiffeners?
         1 $ LAYERS = number of layers (max. = 6)
    N      $ Are all the layers of constant thickness?
         1 $ MATL = type of material for shell wall layer no.( 1)
  11153846 $ G(i) = shear modulus of ith layer, G( 1)
  29000000 $ EX(i)= modulus in meridional direction, EX( 1)
  29000000 $ EY(i)= modulus in circumferential direction, EY( 1)
 0.3000000$ UXY(i)= Poisson's ratio (EY*UXY = EX*UYX).   UXY( 1)
         0 $ ALPHA1(i)=coef. thermal exp. in merid. direction, ALPHA1( 1)
         0 $ ALPHA2(i)=coef. thermal exp. in circ. direction, ALPHA2( 1)
    Y      $ Do you wish to include plasticity in this segment?
    N      $ Do you wish to include creep in this segment?
    N      $ Is this a new shell wall material?
         2 $ NTIN = number of meridional callouts for variable thickness
         2 $ NTYPE = control for meaning of thickness callout (2=z, 3=r)
       480 $ Z(I)  = axial coordinate of Ith thickness callout, z( 1)
      1200 $ Z(I)  = axial coordinate of Ith thickness callout, z( 2)
         1 $ TIN(i) = thickness at Ith callout, TIN( 1)
```

```
        1 $ TIN(i) = thickness at Ith callout, TIN( 2)
    N     $ Do you want to have C(i,j) printed for this segment?
       59 $ NMESH=no. of node points (5=min.;98=max.)SEGMENT NO.( 3)
        1 $ NTYPEH= control integer (1 or 2 or 3) for nodal point spacing
        2 $ NHVALU= number of callouts for nodal point spacing
        1 $ IHVALU(I)= Ith callout for nodal point spacing, IHVALU( 1)
       58 $ IHVALU(I)= Ith callout for nodal point spacing, IHVALU( 2)
       15 $ HVALU=meridional arc length between nodal points,HVALU( 1)
 10.70000$ HVALU=meridional arc length between nodal points,HVALU( 2)
        4 $ NSHAPE= indicator (1,2 or 4) for geometry of meridian
        4 $ NST    = indicator for geometry: 1=general, 4=ellipsoid
 300.7500$ ZMAX   = length of half of axial axis of ellipsoid (p.P9)
 600.7500$ XMAX   = length of half of radial a x i sf ellipsoid
 300.7500$ ZA     = axial coordinate to beginning of segment (p.P9)
 601.4592$ ZB     = axial coordinate to end of segment (p. P9)
        0 $ ALPHAT= radial coordinate from axis of rev. to center.
    Y     $ Do you want to print out the (z,r) coordinates (input data)?
    Y     $ Do you want to print out debugging quantities for geometry?
        1 $ IMP    = indicator for imperfection (0=none, 1=some)
        2 $ ITYPE  = indicator (1 or 2) for type of imperfection
 1.500000$ WO     = amplitude of pure sinusoidal imperfection
       50 $ WLNGTH= half-wavelength of pure sinusoidal imperfection
        3 $ NTYPEZ= control (1 or 3) for reference surface location
0.7500000$ ZVAL   = distance from leftmost surf. to reference surf.
    Y     $ Do you want to print out r(s), r'(s), etc. for this segment?
        0 $ NRINGS= number (max=20) of discrete rings in this segment
    N     $ Do you want general information on loading?
        0 $ NTSTAT = number of temperature callout points along meridian
        1 $ NPSTAT = number of meridional callouts for pressure
        0 $ PN = normal pressure, positive as shown on p. P41.
        0 $ PT = meridional traction, positive along increasing s.
        1 $ ISTEP = control integer for time variation of pressure
    N     $ Do you want to print out distributed loads along meridian?
        0 $ LINTYP=control for line loads or disp.(0=none,1=some)
    N     $ Do you want to include smeared stiffeners?
        1 $ LAYERS = number of layers (max. = 6)
    N     $ Are all the layers of constant thickness?
        1 $ MATL = type of material for shell wall layer no.( 1)
 11153846 $ G(i) = shear modulus of ith layer, G( 1)
 29000000 $ EX(i)= modulus in meridional direction, EX( 1)
 29000000 $ EY(i)= modulus in circumferential direction, EY( 1)
0.3000000$ UXY(i)= Poisson's ratio (EY*UXY = EX*UYX).   UXY( 1)
        0 $ ALPHA1(i)=coef. thermal exp. in merid. direction, ALPHA1( 1)
        0 $ ALPHA2(i)=coef. thermal exp. in circ. direction, ALPHA2( 1)
    Y     $ Do you wish to include plasticity in this segment?
    N     $ Do you wish to include creep in this segment?
    N     $ Is this a new shell wall material?
        3 $ NTIN = number of meridional callouts for variable thickness
        2 $ NTYPE = control for meaning of thickness callout (2=z, 3=r)
        0 $ Z(I)  = axial coordinate of Ith thickness callout, z( 1)
       15 $ Z(I)  = axial coordinate of Ith thickness callout, z( 2)
 299.3728$ Z(I)  = axial coordinate of Ith thickness callout, z( 3)
        1 $ TIN(i) = thickness at Ith callout, TIN( 1)
 1.500000$ TIN(i) = thickness at Ith callout, TIN( 2)
 1.500000$ TIN(i) = thickness at Ith callout, TIN( 3)
    N    $ Do you want to have C(i,j) printed for this segment?
    N    $ Do you want information on time functions for loading?
       1 $ IUTIME = control for time increment (0 or 1). IUTIME
0.3000000E-01 $ DTIME = time increment
 2.000000$ TMAX = maximum time to be encountered during this case
       1 $ NFTIME= number of different functions of time
       2 $ NPOINT=no. of points j for ith load factor F(i,j). i=( 1)
       0 $ T(i,j)=jth time callout for ith time function, j =( 1)
       2 $ T(i,j)=jth time callout for ith time function, j =( 2)
       1 $ F(i,j)=jth value for ith load factor. j =( 1)
       1 $ F(i,j)=jth value for ith load factor. j =( 2)
    N    $ Do you want general information on constraint conditions?
       0 $ Number of poles (places where r=0) in SEGMENT( 1)
       1 $ At how many stations is this segment constrained to ground?
       1 $ INODE = nodal point number of constraint to ground, INODE( 1)
       1 $ IUSTAR=axial displacement constraint (0 or 1 or 2)
       1 $ IVSTAR= circumferential displacement (0=free, 1=constrained)
       1 $ IWSTAR=radial displacement(0=free,1=constrained,2=imposed)
       1 $ ICHI=meridional rotation (0=free,1=constrained,2=imposed)
       0 $ D1    = radial component of offset of ground support
       0 $ D2    = axial component of offset of ground support
    Y    $ Is this constraint the same for both prebuckling and buckling?
    N    $ Is this segment joined to any lower-numbered segments?
       0 $ Number of poles (places where r=0) in SEGMENT( 2)
       0 $ At how many stations is this segment constrained to ground?
    Y    $ Is this segment joined to any lower-numbered segments?
       1 $ At how may stations is this segment joined to previous segs.?
       1 $ INODE = node in current segment (ISEG) of junction, INODE( 1)
       1 $ JSEG  = segment no. of previous segment involved in junction
      33 $ JNODE = node in previous segmnt (JSEG) of junction
       1 $ IUSTAR= axial displacement (0=not slaved, 1=slaved)
       1 $ IVSTAR= circumferential displacement (0=not slaved, 1=slaved)
       1 $ IWR= radial displacement (0=not slaved, 1=slaved)
    1 $ ICHI  = meridional rotation (0=not slaved, 1=slaved)
    0 $ D1    = radial component of juncture gap
    0 $ D2    = axial component of juncture gap
Y    $ Is this constraint the same for both prebuckling and buckling?
       0 $ Number of poles (places where r=0) in SEGMENT( 3)
       1 $ At how many stations is this segment constrained to ground?
      59 $ INODE = nodal point number of constraint to ground, INODE( 1)
       0 $ IUSTAR=axial displacement constraint (0 or 1 or 2)
       0 $ IVSTAR= circumferential displacement (0=free, 1=constrained)
       0 $ IWSTAR=radial displacement(0=free,1=constrained,2=imposed)
       0 $ ICHI=meridional rotation (0=free,1=constrained,2=imposed)
       0 $ D1    = radial component of offset of ground support
       0 $ D2    = axial component of offset of ground support
    Y    $ Is this constraint the same for both prebuckling and buckling?
    Y    $ Is this segment joined to any lower-numbered segments?
       1 $ At how may stations is this segment joined to previous segs.?
       1 $ INODE = node in current segment (ISEG) of junction, INODE( 1)
       2 $ JSEG  = segment no. of previous segment involved in junction
      61 $ JNODE = node in previous segmnt (JSEG) of junction
       1 $ IUSTAR= axial displacement (0=not slaved, 1=slaved)
```

```
1 $ IVSTAR= circumferential displacement (0=not slaved, 1=slaved)
1 $ IWSTAR= radial displacement (0=not slaved, 1=slaved)
1 $ ICHI  = meridional rotation (0=not slaved, 1=slaved)
0 $ D1    = radial component of juncture gap
0 $ D2    = axial component of juncture gap
Y $ Is this constraint the same for both prebuckling and buckling?
N $ Given existing constraints, are rigid body modes possible?
```

Table A.16 Required Data to Start the Execution of B5AMES Main Processor (Static Analysis)

```
   0 $ INDIC = analysis type indicator (0 or -2 or -3). INDIC
   N $ Type HELP or H for more information on INDIC, N for "no more"
   1 $ IDEFORM=indicator (0 or 1) for type of plasticity theory
   1 $ ICPRE = control (0 or 1) for type of eigenvalue problem
   0 $ KSTEP = starting time step number
  12 $ KMAX = maximum (less than 49) time step number
  20 $ MAXTRL =  maximum number of trials at each load level
  20 $ ITMAX = maximum number of Newton iterations for each trial
   0 $ ITIME = control (0 or 1) for time increments
   0 $ TIME = starting time (need not be zero in initial run)
   Y $ Do you want stations where plastic flow occurs listed?
   1
 100
```

Table A.17 Required Data to Re-Start the Execution of B5AMES Main Processor (Static Analysis)

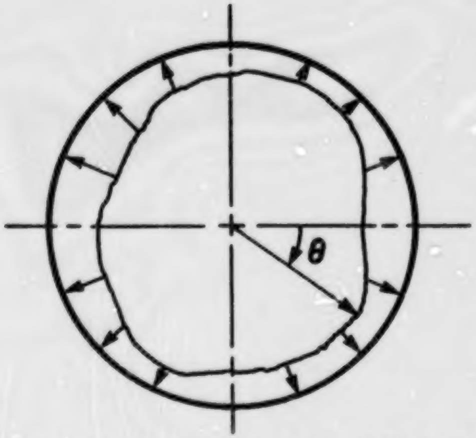
```
   0 $ INDIC = analysis type indicator (0 or -2 or -3). INDIC
   N $  Type HELP or H for more information on INDIC, N for "no more"
   1 $ IDEFORM=indicator (0 or 1) for type of plasticity theory
   1 $ ICPRE = control (0 or 1) for type of eigenvalue problem
  12 $ KSTEP = starting time step number
  13 $ KMAX = maximum (less than 49) time step number
  20 $ MAXTRL =  maximum number of trials at each load level
  20 $ ITMAX = maximum number of Newton iterations for each trial
   1 $ ITIME = control (0 or 1) for time increments
 .33 $ TIME = starting time (need not be zero in initial run)
.005 $ DTIME = new time increment
   Y $ Do you want stations where plastic flow occurs listed?
   1
 100
```

Table A.18 Input Data Required to Execute B5AMES Post Processor

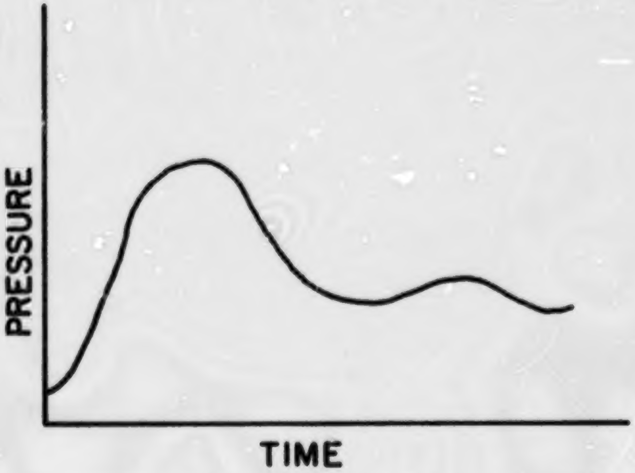
```
   1 $ IPRINT = control (1 or 2 or 3) for list output
   0 $ IPLOT  = control (0 or 1 or 2 or 3) for plot output
  -1 $ NLAST = plot options  (-1=none, 0=geometry, 1=u,v,w)
   N $ Do you want to plot the discretized model?
   4 $ NSTEPS = quantity of time steps for list or plot output
  10 $ LSTEP(i)= output is desired for time step no.
  11 $ LSTEP(i)= output is desired for time step no.
  12 $ LSTEP(i)= output is desired for time step no.
  13 $ LSTEP(i)= output is desired for time step no.
 .27 $ TIME(i) = time corresponding to LSTEP( 1)
 .30 $ TIME(i) = time corresponding to LSTEP( 2)
 .33 $ TIME(i) = time corresponding to LSTEP( 3)
 .335$ TIME(i) = time corresponding to LSTEP( 4)
   1 $ IFLAG = control (0 or 1) for amount of prebuckling output
   4 $ FMAX = normalization factor for plotting deformed shell
   N $ Do you want prebuckling output for all the segments?
   3 $ NSEG = total number of segments in the structure
   Y $ Do you want any prebuckling output for segment no.( 1)
   N $ Do you want output for all nodal points in seg. no.( 1)
   1 $ IMIN = minimum nodal point number for output, Seg.( 1)
  10 $ IMAX = maximum nodal point number for output, Seg.( 1)
  Y  $ Do you want any prebuckling output for segment no.( 2)
  Y  $ Do you want output for all nodal points in seg. no.( 1)
  N  $ Do you want any prebuckling output for segment no.( 3)
```

Table A.19 Input Data Required to Execute B5AMES Main Processor
(Stability Analysis)

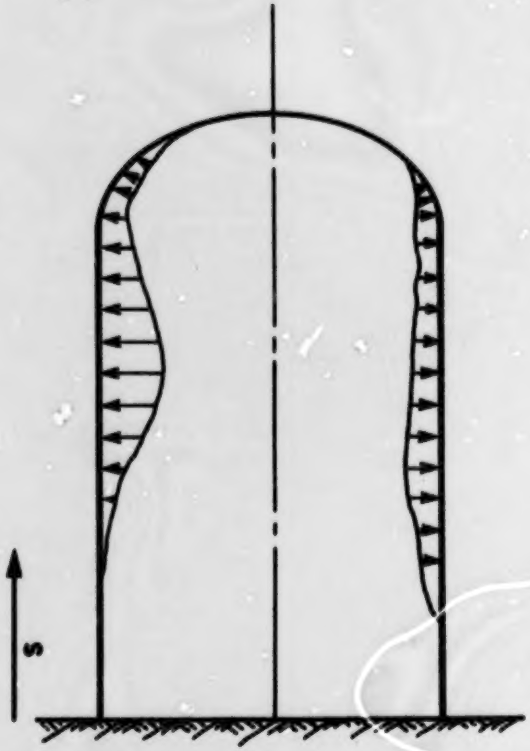
```
 -3 $ INDIC = analysis type indicator (0 or -2 or -3). INDIC
 N  $ Type HELP or H for more information on INDIC, N for "no more"
 1  $ IDEFORM=indicator (0 or 1) for type of plasticity theory
 1  $ ICPRE = control (0 or 1) for type of eigenvalue problem
 4  $ NSTEPS = quantity of time steps for stability calculation
 10 $ LSTEPS(i)=time step number for buckling, Step no.( 1)
 11 $ LSTEPS(i)=time step number for buckling, Step no.( 2)
 12 $ LSTEPS(i)=time step number for buckling, Step no.( 3)
 13 $ LSTEPS(i)=time step number for buckling, Step no.( 4)
.27 $ TIMES(i)=time corresponding to LSTEP(i), Step no.( 1)
.30 $ TIMES(i)=time corresponding to LSTEP(i), Step no.( 2)
.33 $ TIMES(i)=time corresponding to LSTEP(i), Step no.( 3)
.335$ TIMES(i)=time corresponding to LSTEP(i), Step no.( 4)
  0 $ NOB   = starting number of circ. waves (buckling analysis)
  0 $ NMINB = minimum number of circ. waves  (buckling analysis)
 50 $ NMAXB = maximum number of circ. waves  (buckling analysis)
  5 $ INCRB = increment in number of circ. waves (buckling)
  Y $ Do you want stations where plastic flow occurs listed?
  1
  100
```



(b) Circumferential Variation, p(t,θ)



(c) Time Variation, p(t)



(a) Meridional Variation, p(t,s)

Figure 2.1 Pressure Variation Within a Containment Vessel

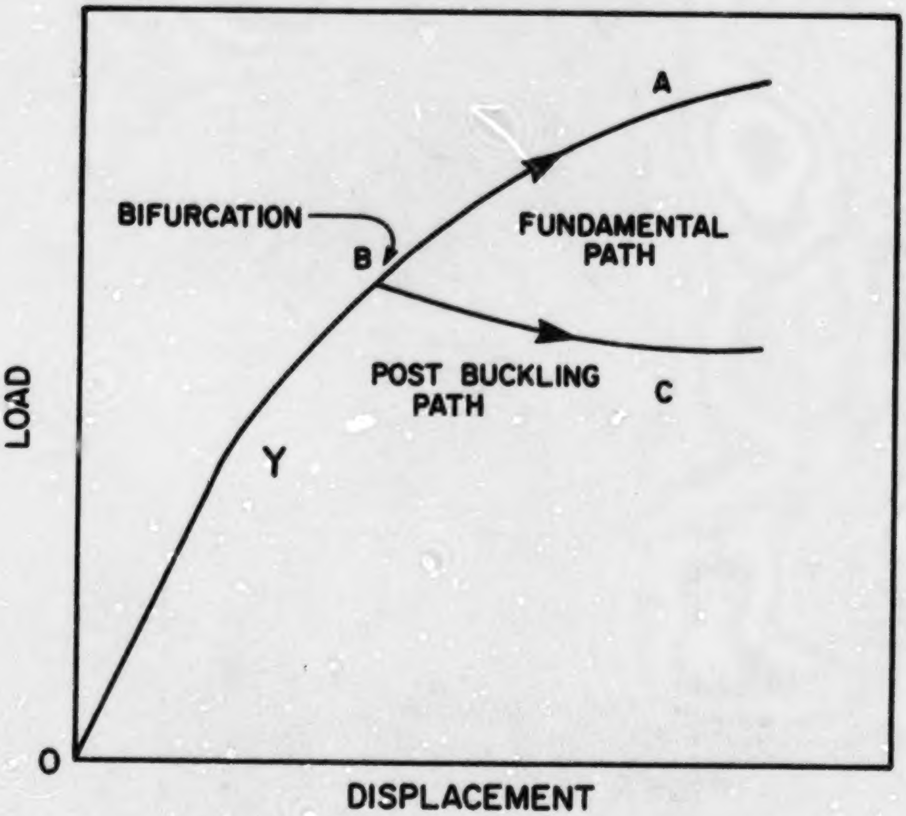

A
BIFURCATION
B
FUNDAMENTAL PATH
POST BUCKLING PATH
C
Y
LOAD
O
DISPLACEMENT

Figure 3.1 Fundamental and Postbuckling Load-Displacement Paths

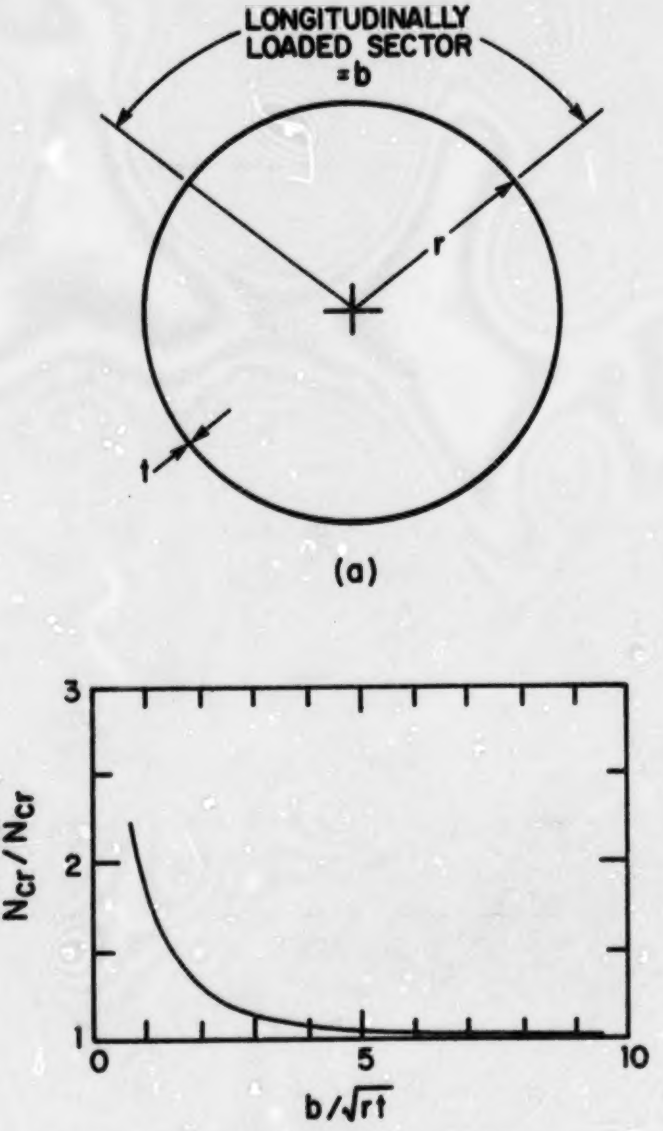


Figure 3.2 Effect of Nonaxisymmetric Longitudinal Distribution on Critical Stress [3.15]

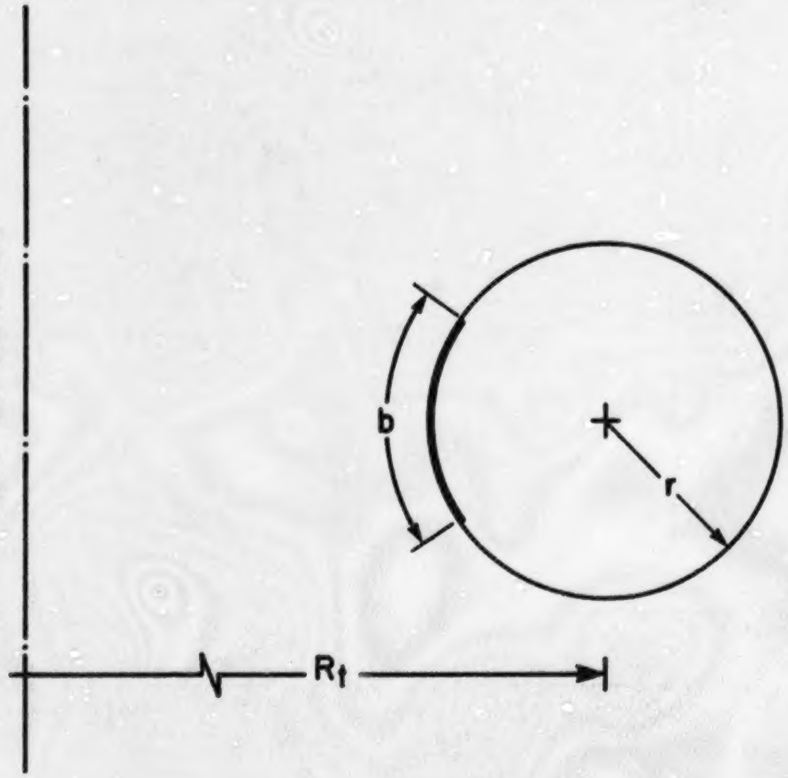

AXIS OF ROTATION
b
r
R_t

Figure 3.3 Torus Model for Buckling of Axisymmetric Shell with Nonaxisymmetric Stress-Resultant

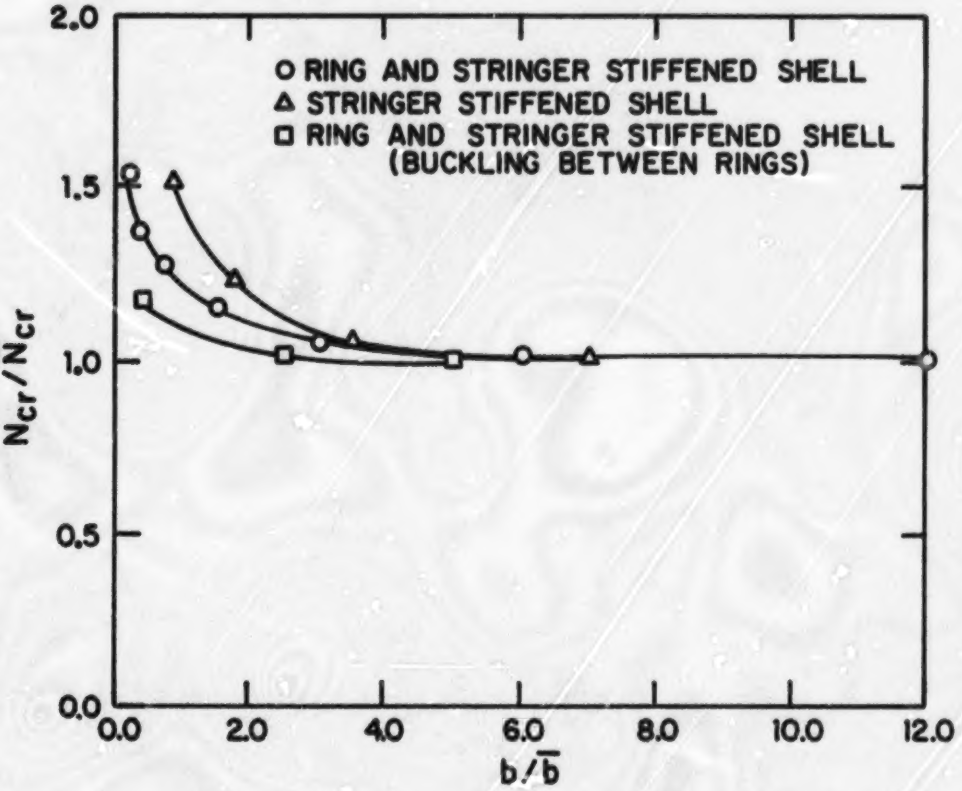
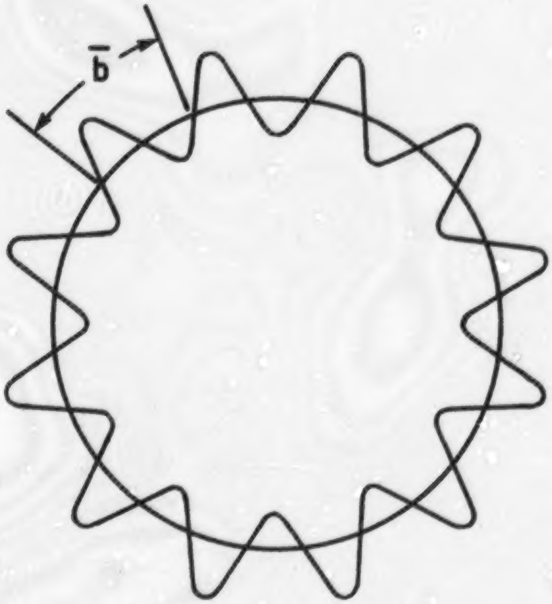
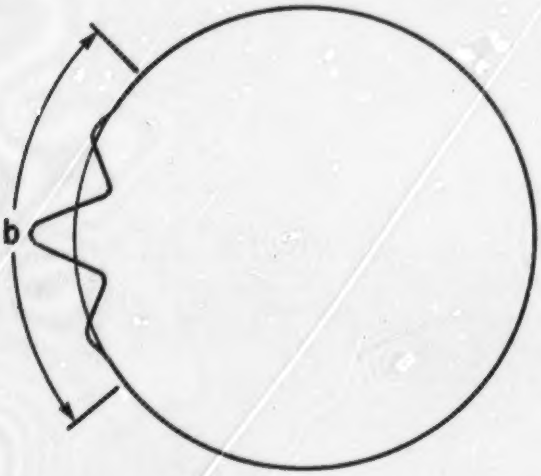


Figure 3.4 Effect of Circumferentially Varying Longitudinal Stress on Critical Load for Stiffened Cylinders [3.9]

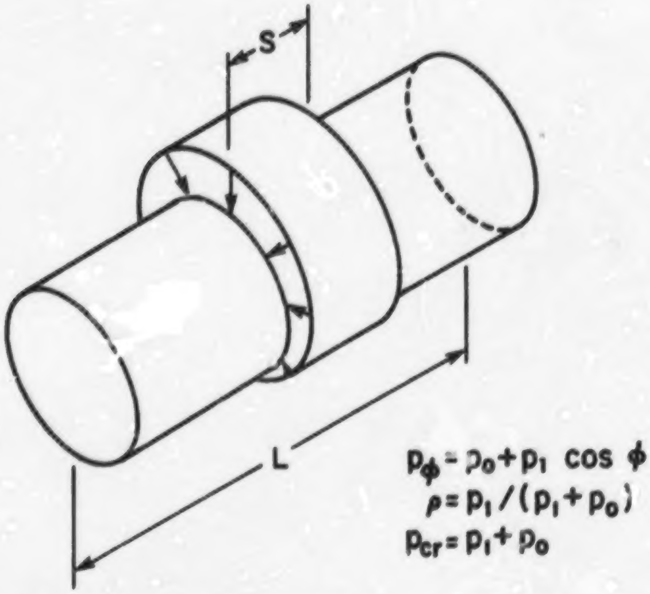


(a) BUCKLING PATTERN FOR UNIFORM AXIAL COMPRESSION

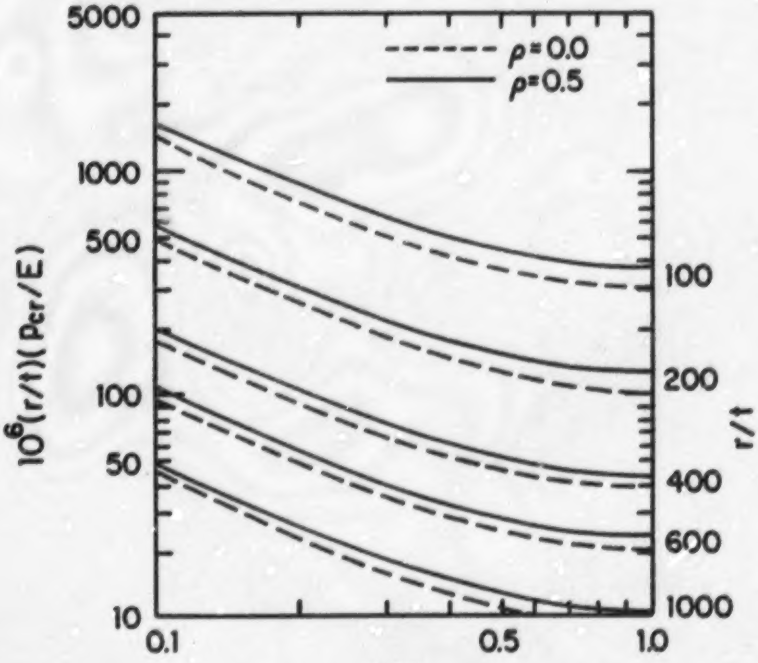


(b) BUCKLING PATTERN WITH UNIFORM COMPRESSION OVER ARC LENGTH b

Figure 3.5 Buckling Patterns of Axisymmetrically and Nonaxisymmetrically Loaded Cylinder

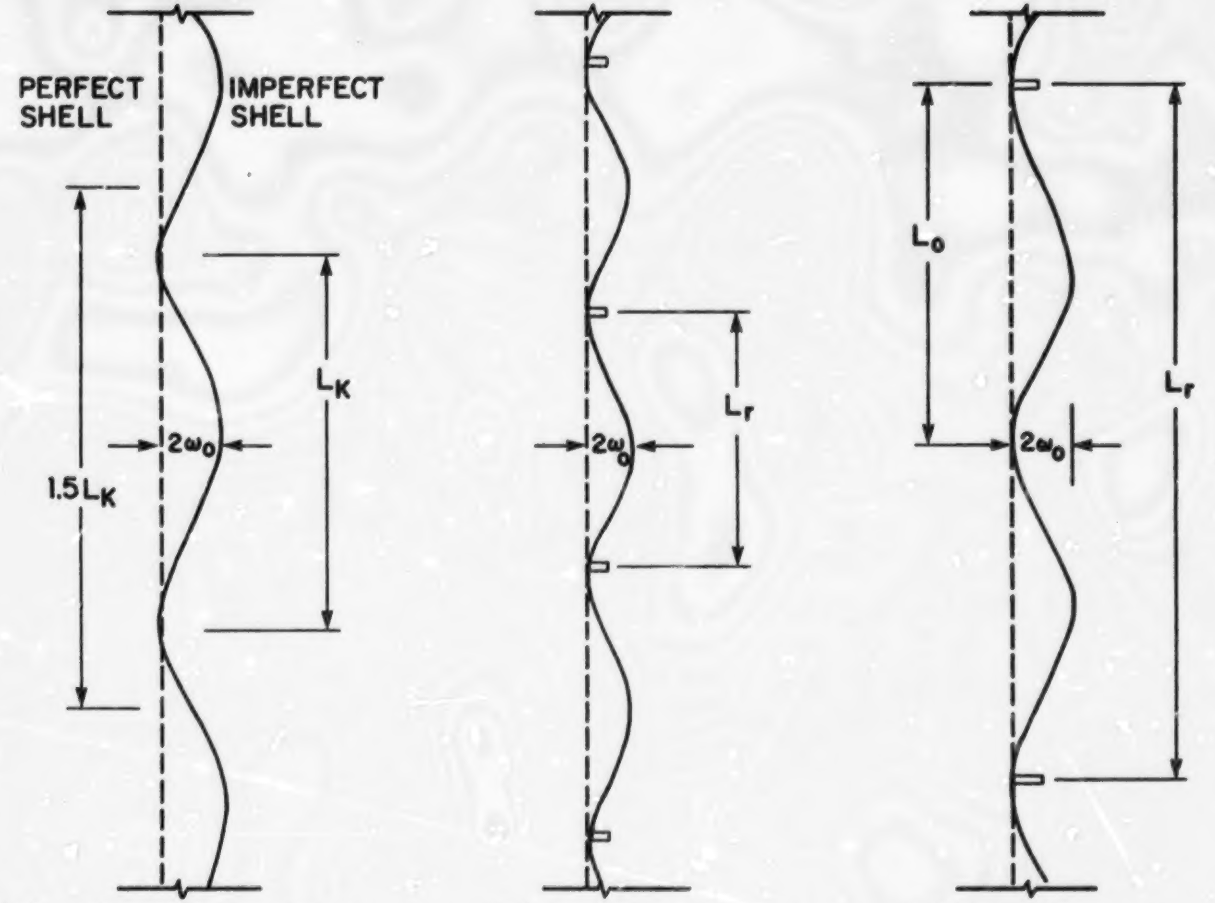

S
L
$p_\phi = p_0 + p_1 \cos\phi$
$\rho = p_1/(p_1 + p_0)$
$p_{cr} = p_1 + p_0$

(a) CIRCUMFERENTIALLY VARYING RADIAL PRESSURE



(b) BUCKLING PRESSURE

Figure 3.6 Effect of Circumferential Pressure Variation [3.18]


PERFECT SHELL
IMPERFECT SHELL
1.5 L_K
L_K
$2\omega_o$
L_r
$2\omega_o$
L_o
L_r
$2\omega_o$

(a) UNSTIFFENED SHELL $L_o = L_K$

(b) $L_r < 1.5 L_K$ $L_o = L_r$

(c) $L_r > 1.5 L_K$ $L_o = \mathrm{MIN}(L_r/2, L_K)$

Figure 3.7 Configuration of Axisymmetric Imperfection in BOSOR5 Buckling Analysis [3.3] $L_K = 3.5\sqrt{rt}$

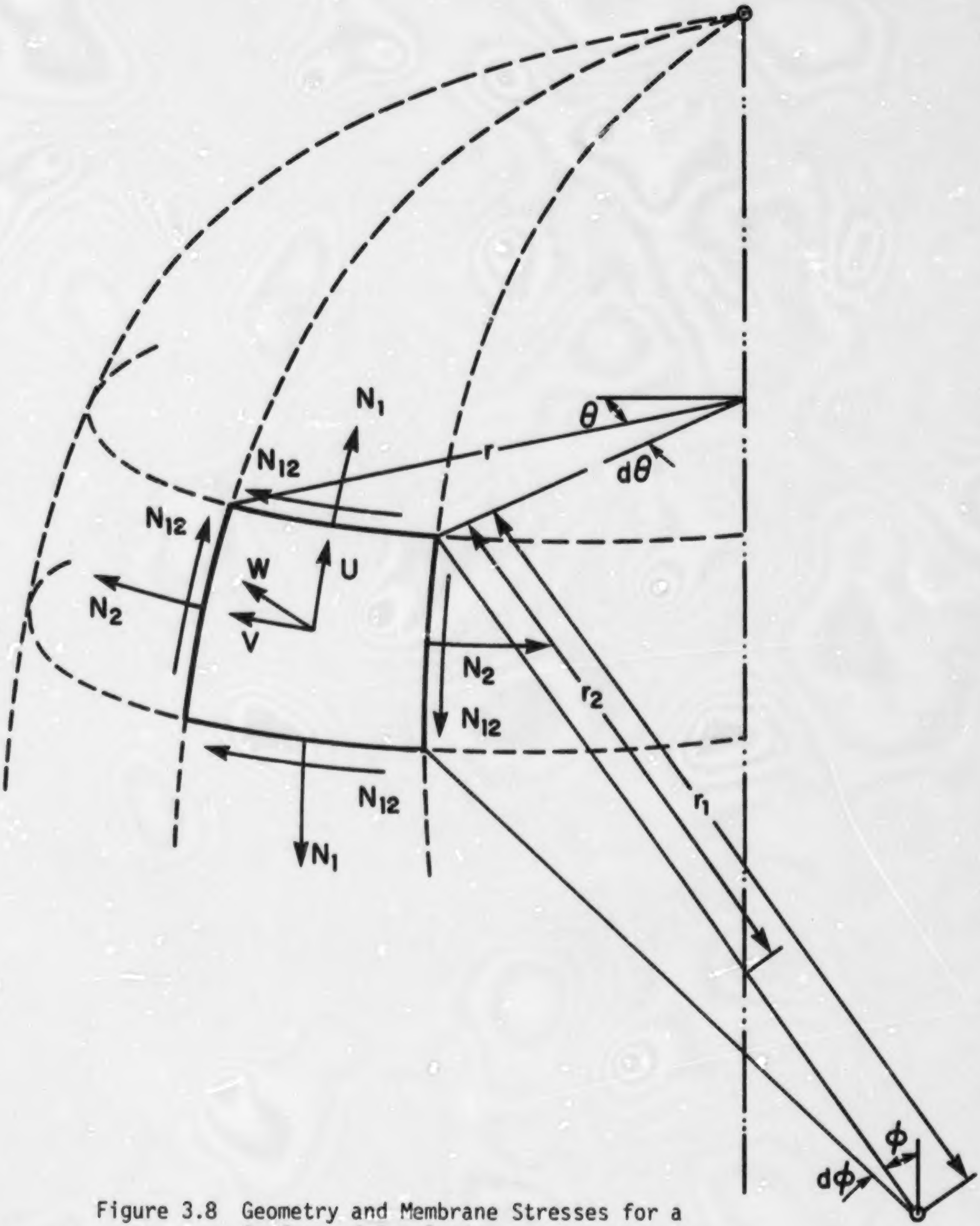

N_1
θ
N_12
r
dθ
N_12
U
W
N_2
V
N_2
r_2
N_12
N_12
r_1
N_1
φ
dφ

Figure 3.8 Geometry and Membrane Stresses for a Surface of Revolution

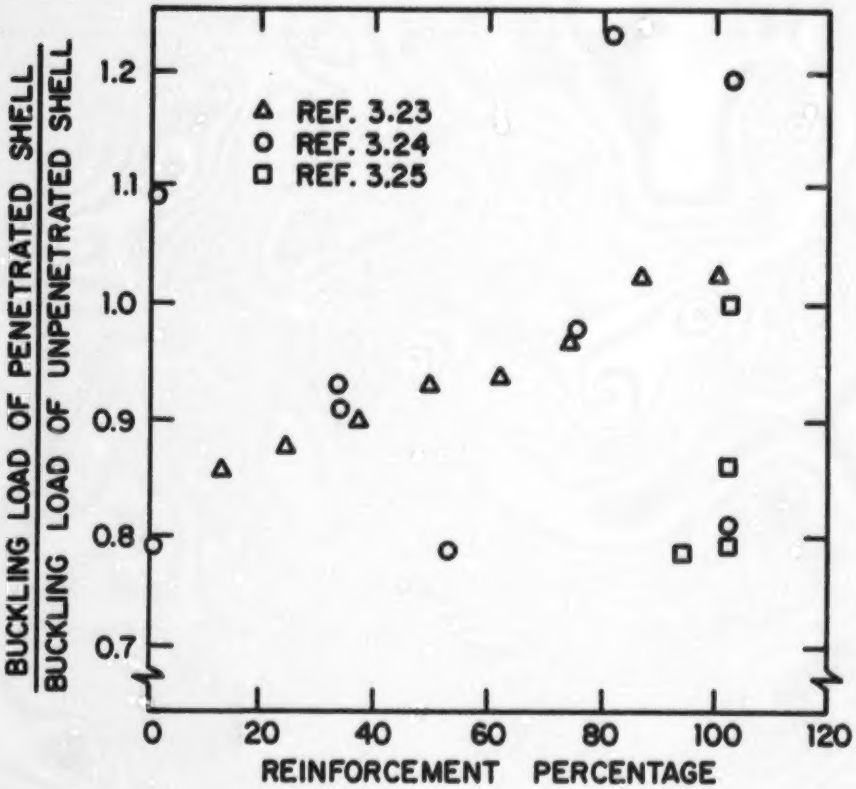


Figure 3.9 Effect of Reinforcement Percentage on the Penetrated Shell Critical Load

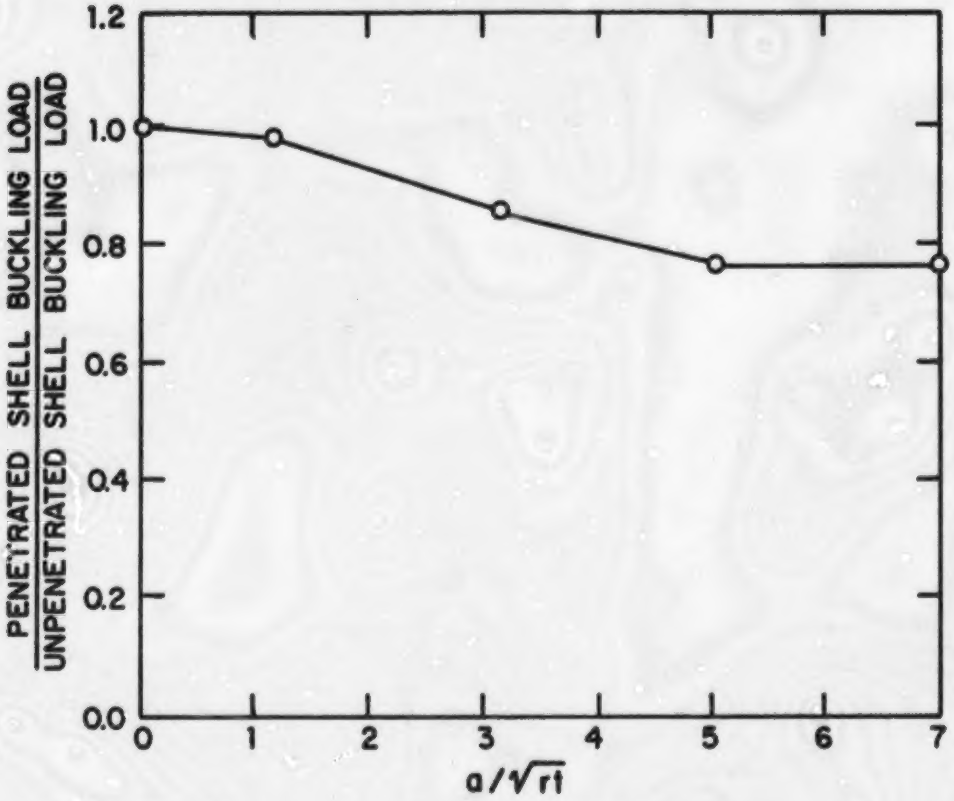


Figure 3.10 Effect of Penetration Size on Shell Critical Load [3.26]

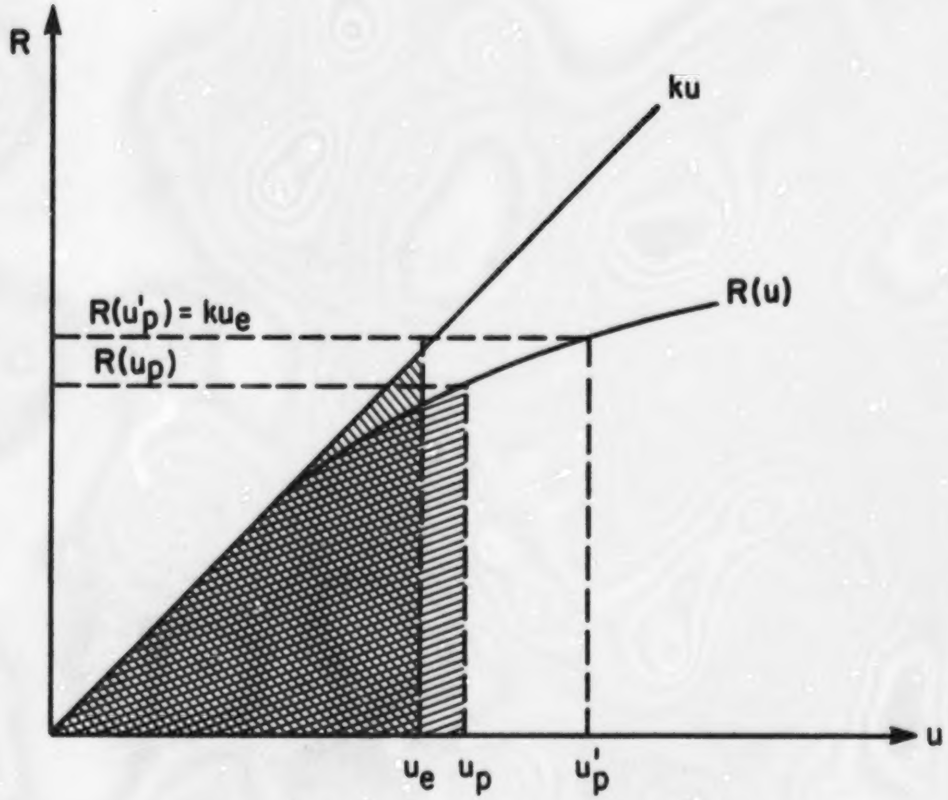

R
ku
R(u)
$R(u'_p) = ku_e$
$R(u_p)$
u
u_e
u_p
u'_p

Figure 3.11 General Nonlinear Single Degree of Freedom System

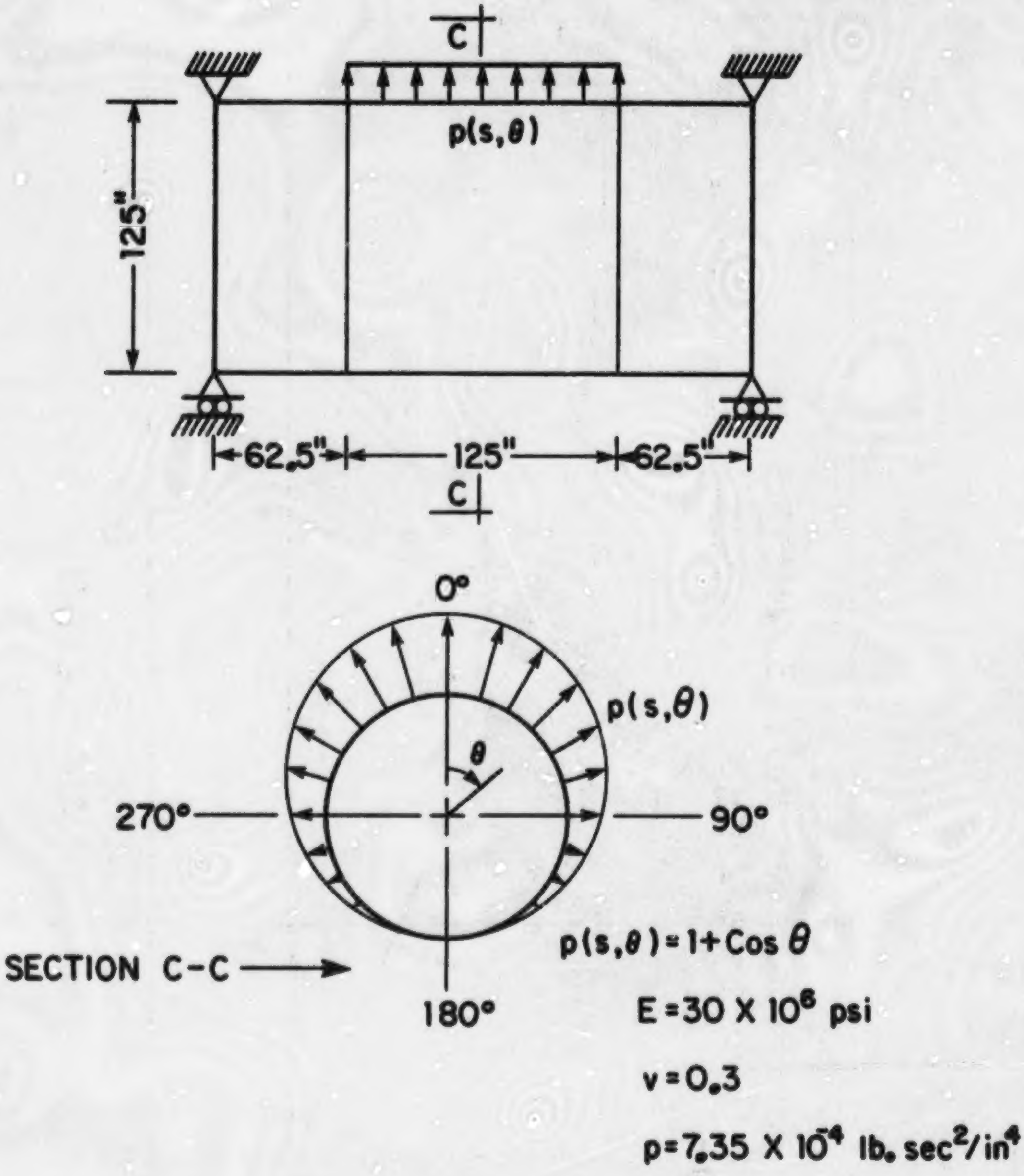

C
p(s,θ)
125"
62.5"
125"
62.5"
C
0°
p(s,θ)
θ
270°
90°
SECTION C-C
180°
p(s,θ) = 1+Cos θ
E = 30 X 10^6 psi
v = 0.3
p = 7.35 X 10^-4 lb. sec^2/in^4

Figure 5.1 Nonaxisymmetric Loaded Shell Used in Sec. 5.4.2 (Problem A)

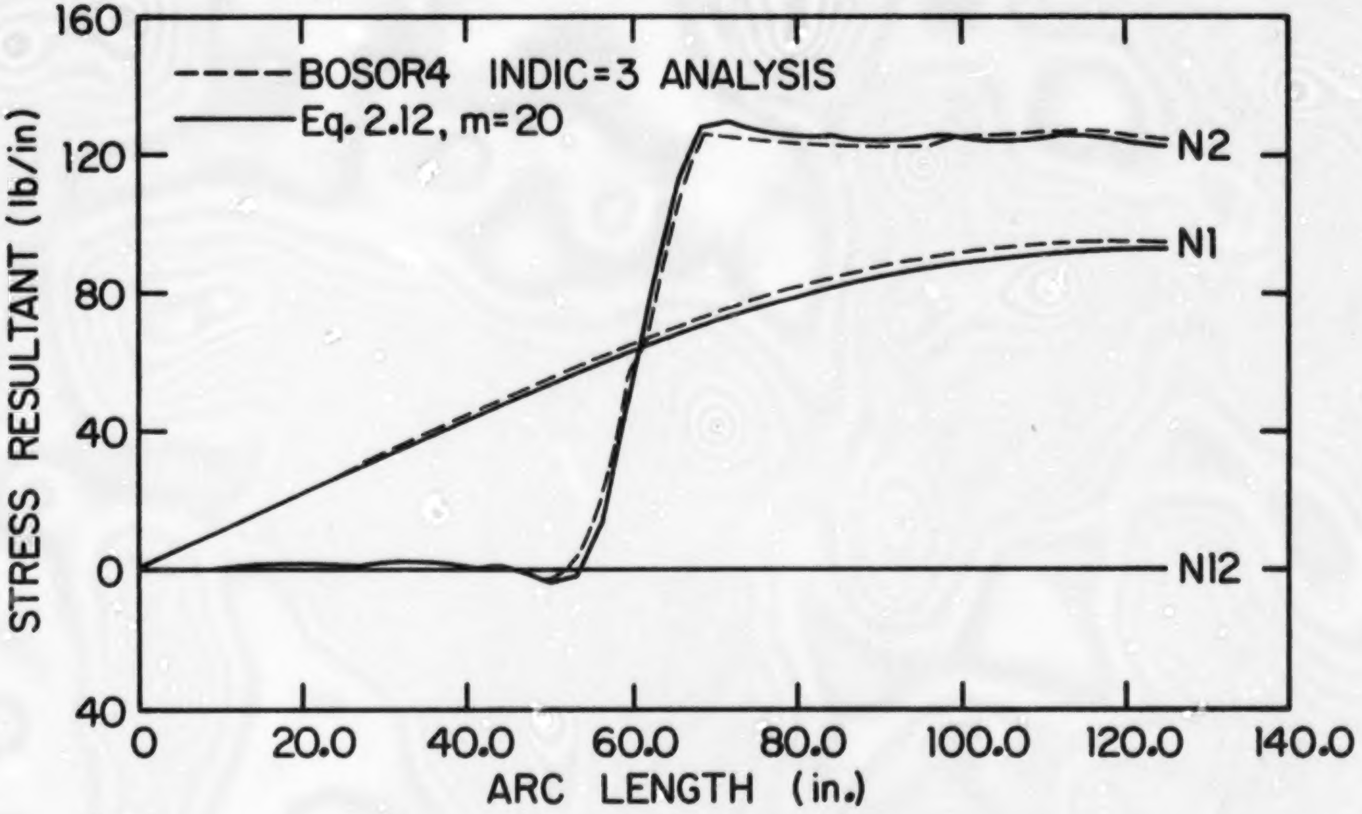


Figure 5.2 Membrane Stress Resultants for Cylinder in Figure 5.1 (θ = 0°)

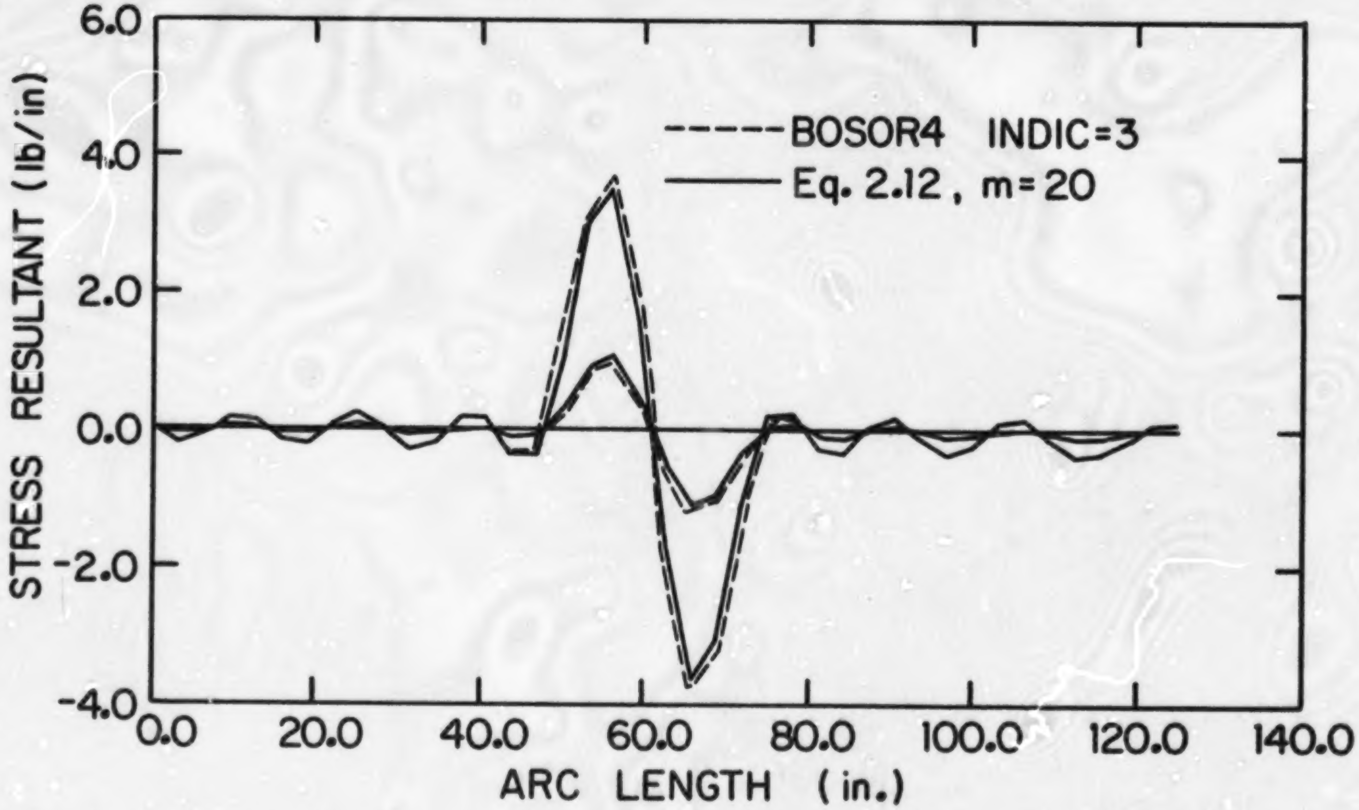


Figure 5.3 Moment Stress Resultants for Cylinder in Figure 5.1 (θ = 0°)

130

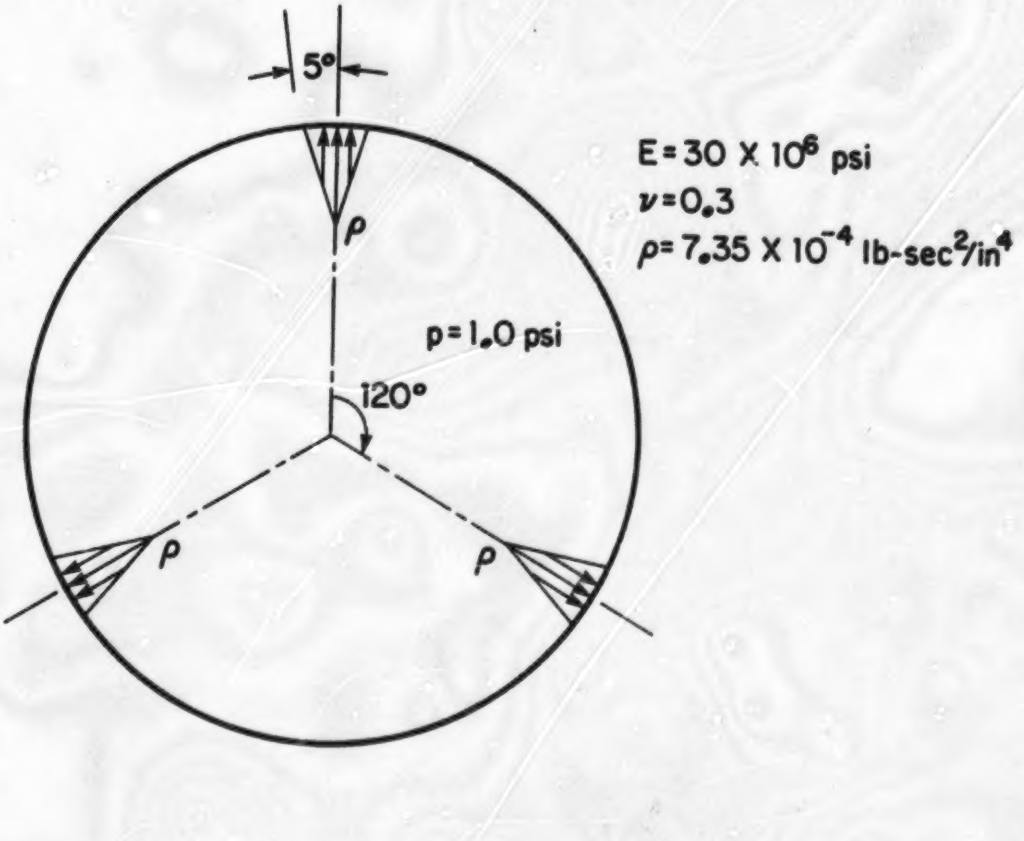

5°
ρ
ρ
ρ
E = 30 X 10^6 psi
ν = 0.3
ρ = 7.35 X 10^-4 lb-sec²/in⁴
p = 1.0 psi
120°

Figure 5.4 Nonaxisymmetric Pressure Distribution Used in Sec. 5.4.2.2

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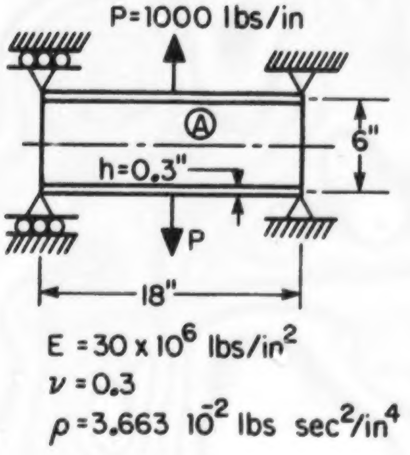
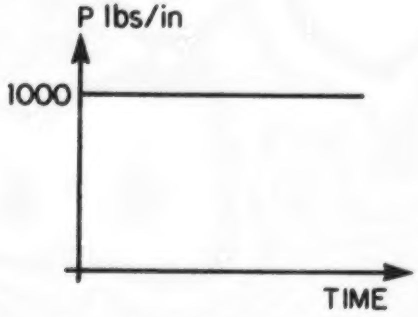
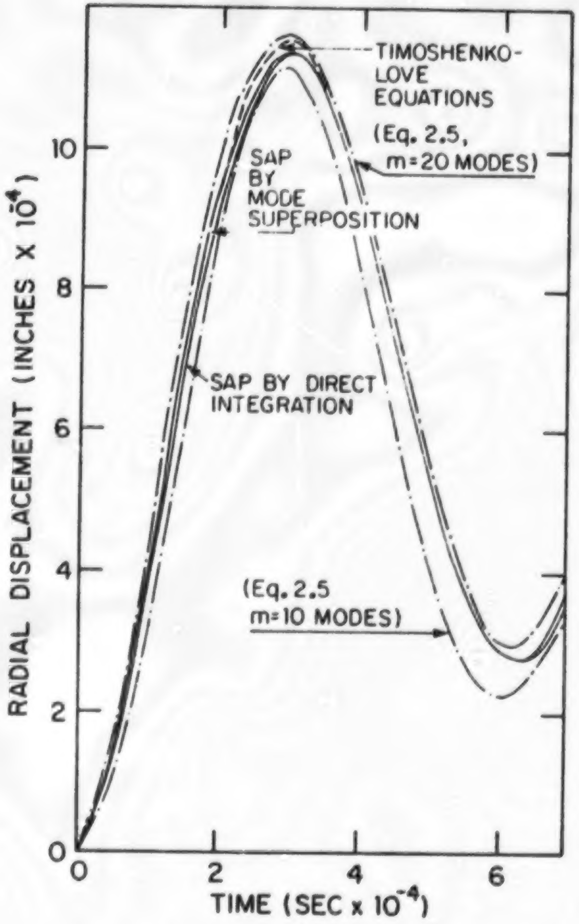

P=1000 lbs/in
Ⓐ
6"
h=0.3"
P
18"
E = 30 x 10⁶ lbs/in²
ν = 0.3
ρ = 3.663 10⁻² lbs sec²/in⁴

a) CYLINDRICAL TUBE



b) TIME VARIATION OF LOAD



c) RADIAL DISPLACEMENT VERSES TIME

Figure 5.5 Response History Analysis of Cylindrical Tube (Sec. 5.4.3)

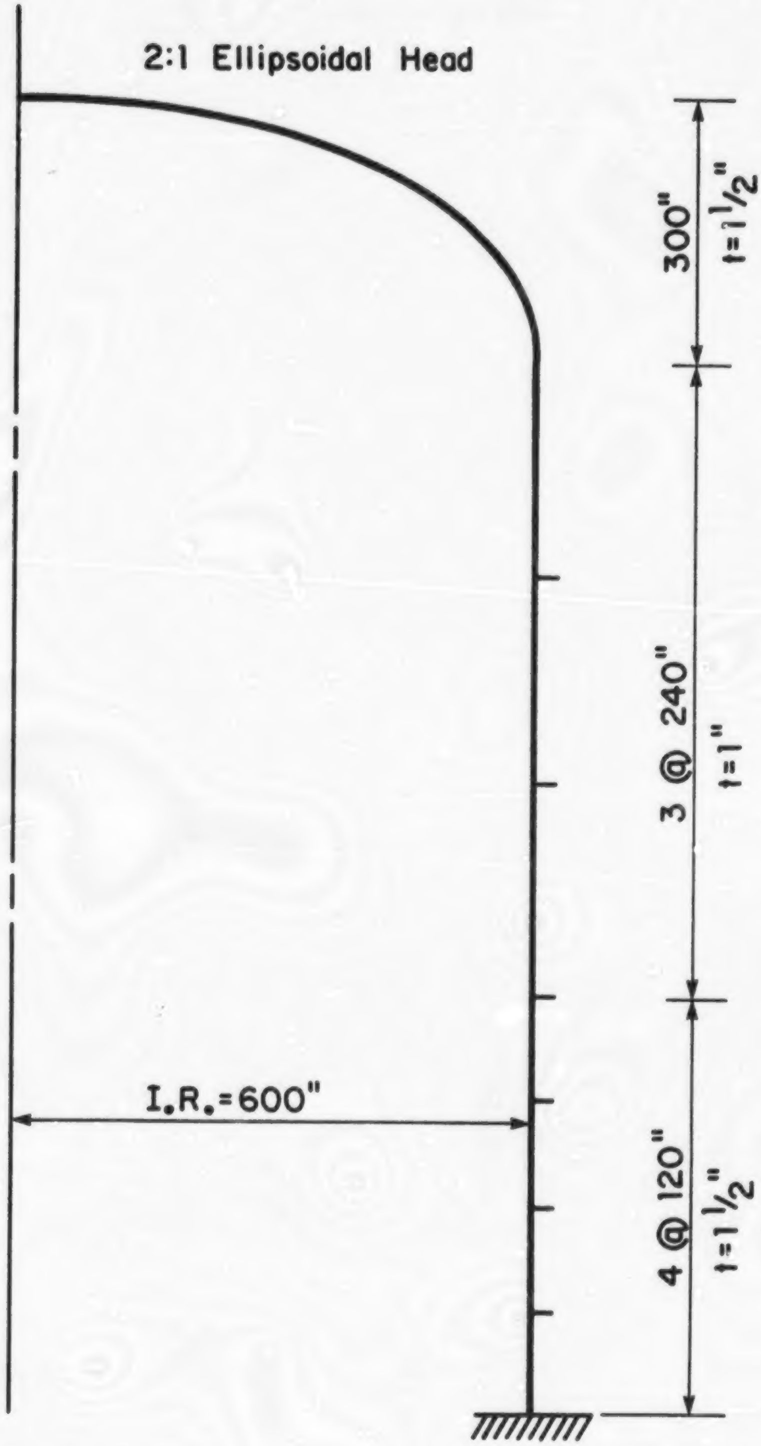
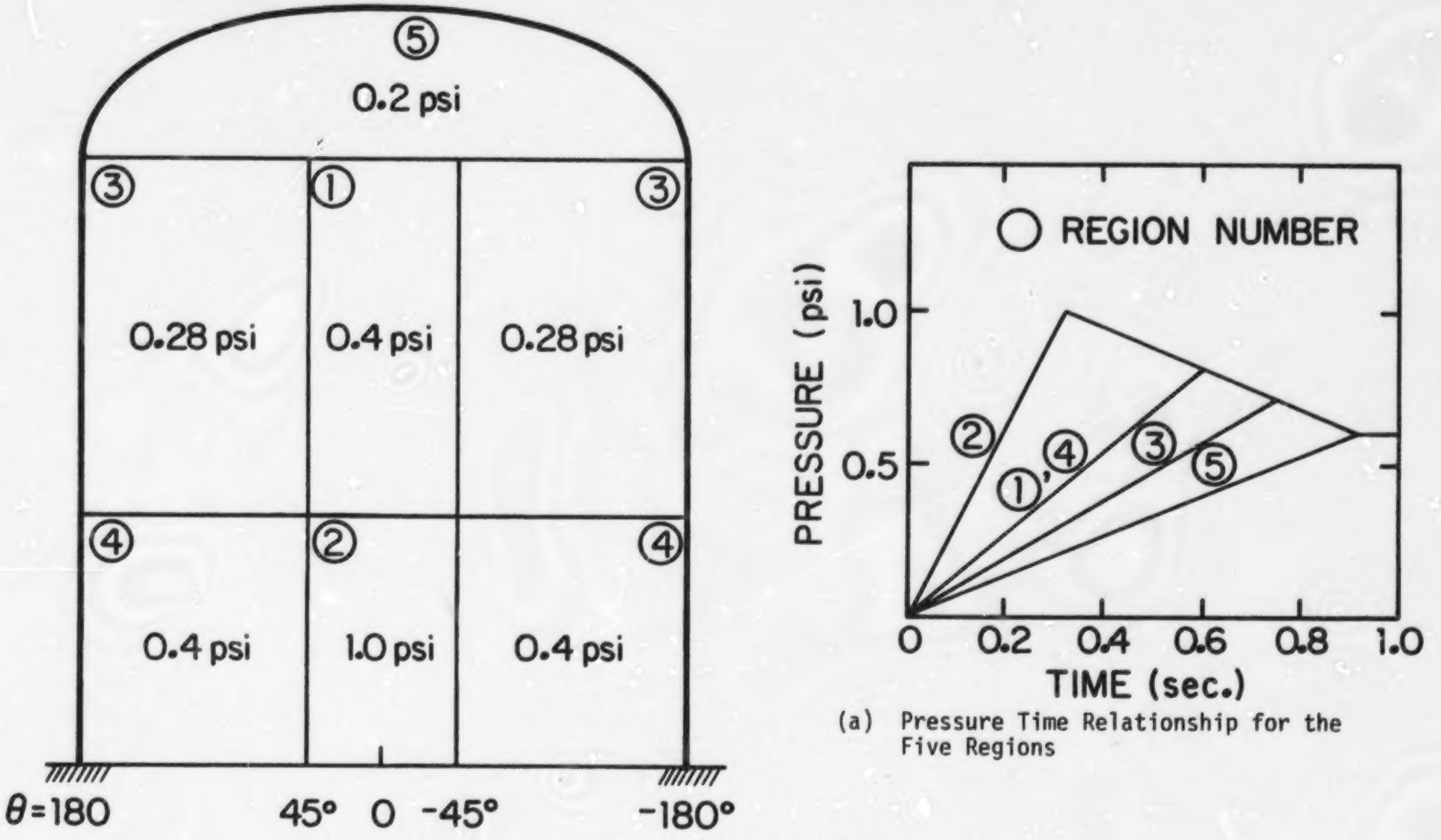

2:1 Ellipsoidal Head
300"
t=1 1/2"
3 @ 240"
t=1"
I.R.=600"
4 @ 120"
t=1 1/2"

Figure 5.6 Containment Vessel Geometry



(a) Pressure Time Relationship for the Five Regions

(b) Pressure Circumferential Distribution in the Five Regions at Time = 0.3 Sec.

Figure 5.7 Pressure Transients Used in the Containment Vessel Analysis in Sec. 5.5

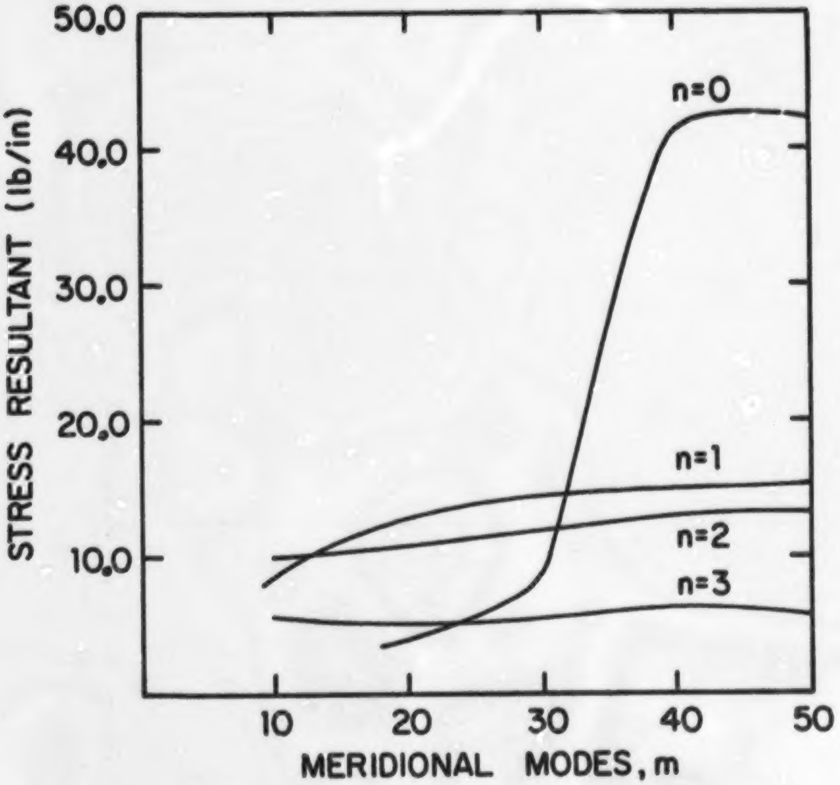


Figure 5.8 Effects of the Meridional Modes on the Maximum Circumferential Stress Resultants in Region Two (0.3 Sec.)

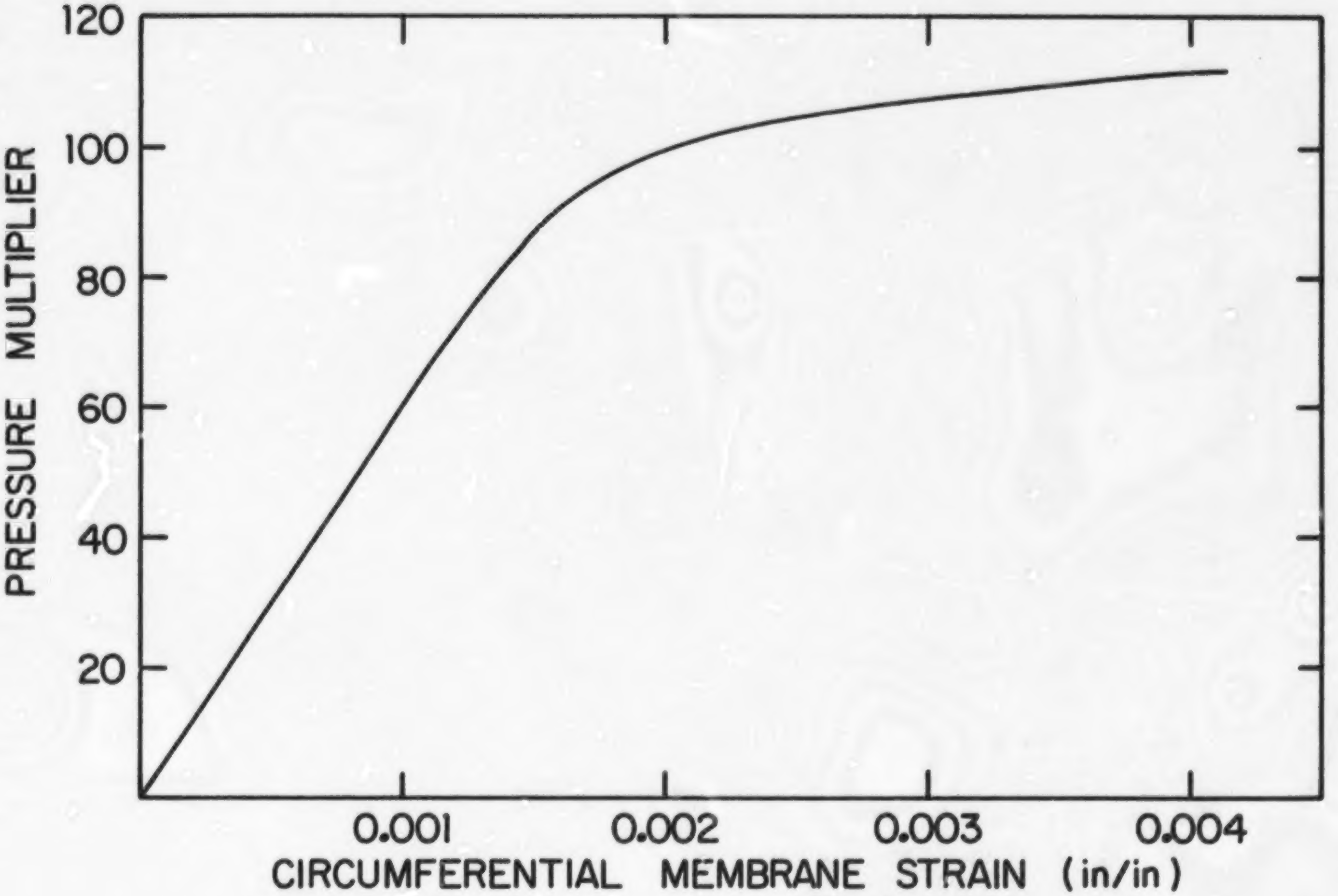


Figure 5.9 Circumferential Membrane Strain at θ = zero and Between Lower Rings for the Containment Shown in Figure 5.6

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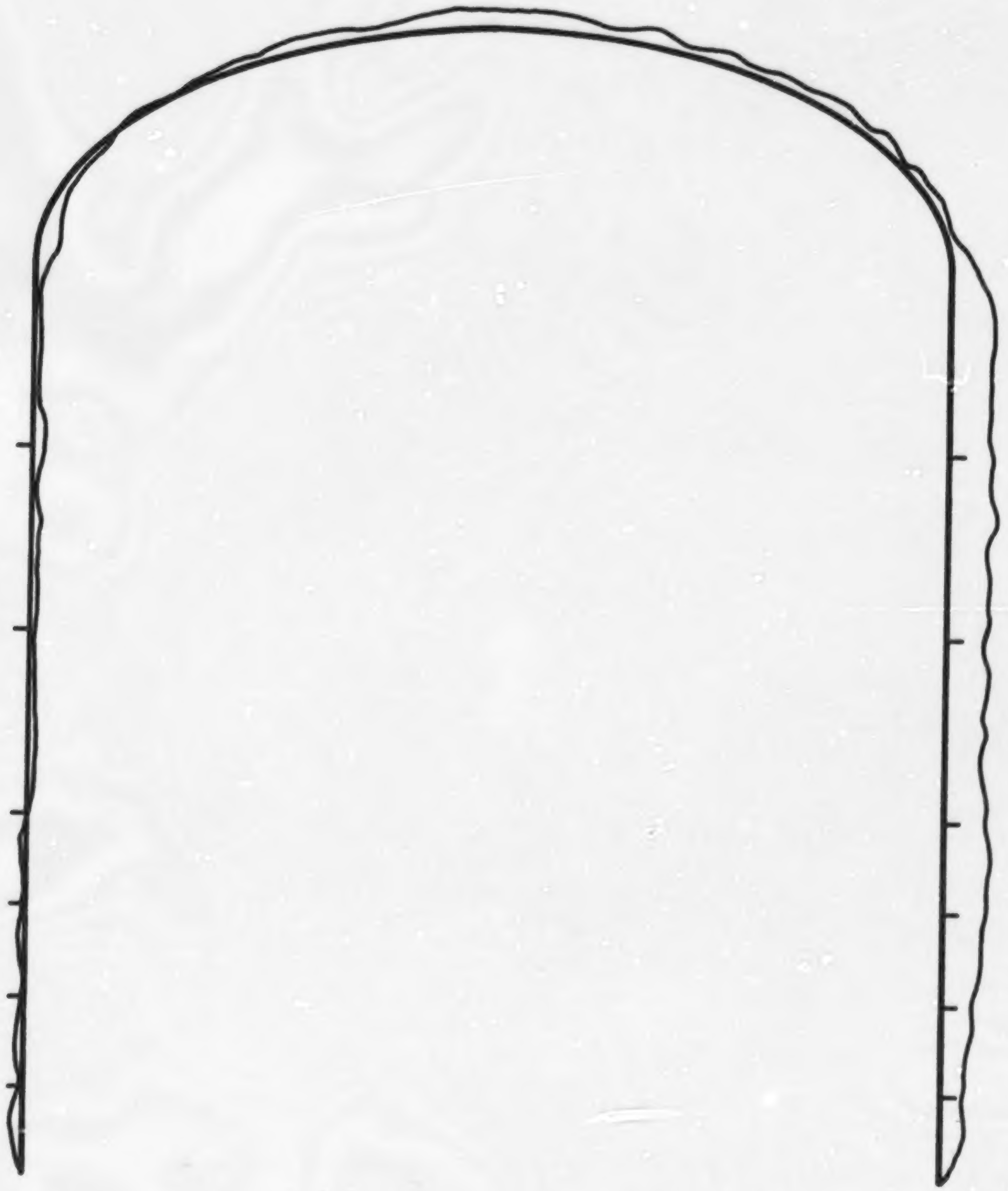

Figure 5.10 Containment Deformed Shape at Time = 0.3 Sec.

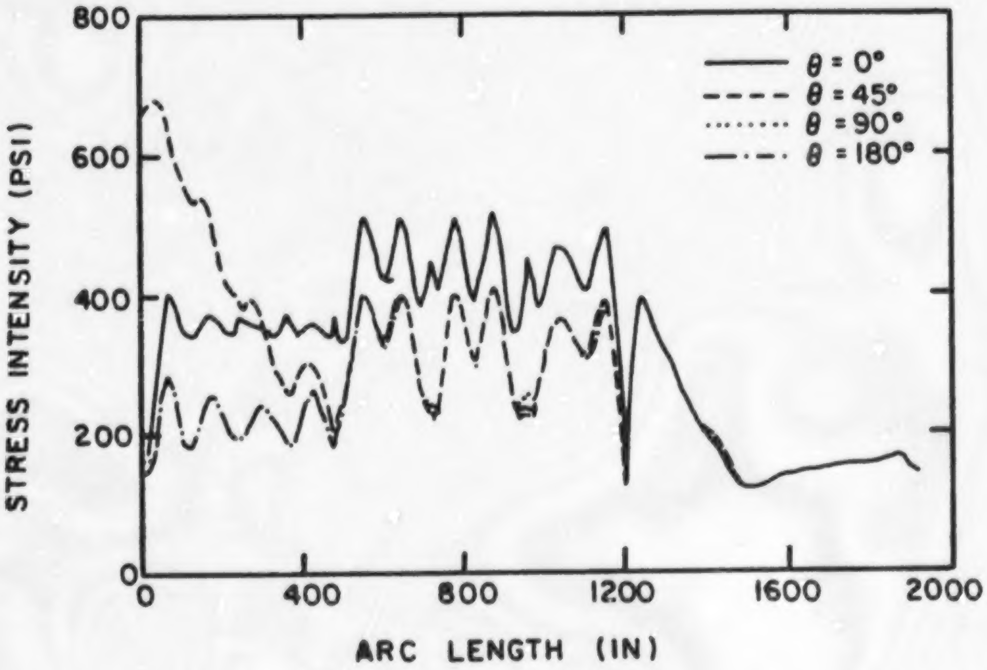

Figure 5.11 Envelope of the Stress Intensities at the Four Meridional Locations

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13. ABSTRACT (200 words or less)

The objective of this work is to develop a methodology for calculating the ultimate capacity of steel containments under very general global or local, uniform or spatially varying dynamic loadings. An axisymmetric geometric model is used and the circumferential variation of the loads is expressed in terms of Fourier series components. The solution, i.e., displacement and stress resultants at any point of time, are obtained and also represented by Fourier series components. Values of these components at discrete locations are evaluated using modal superposition. Failure, i.e., leakage of the containment, is defined to occur when either the membrane strain in the shell reaches twice the shell material yield strain or a bifurcation point occurs. The ASME Boiler and Pressure Vessel Codes Level C criteria can also be applied as a failure condition. A computer program called LOADS was written to evaluate the point-in-time displacements and stress resultants. This program uses the results obtained from a modified BOSOR4 modal analysis program. The results of the LOADS program are input into a modified BOSOR5 computer code to evaluate the containment resistance which considers large deformation and material nonlinear behavior. A user's manual for the BOSOR4/LOADS/BOSOR5 program set is presented.

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